Math 487
Extra Credit 2 **Name:**________________________________

Due: Monday, March 28th

Given that A is the matrix of an isometry, let X and Y be points in $\mathbb{E}$. Suppose that $AX = X'$ and $AY = Y'$. Since A is an isometry, $d(X, Y) = d(X', Y')$. Complete the missing steps in the proof of **Proposition 3.7** by proving that if:

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$, $X' = \begin{bmatrix} x_1' \\ x_2' \\ 1 \end{bmatrix}$, $Y = \begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix}$, and $y = \begin{bmatrix} y_1' \\ y_2' \\ 1 \end{bmatrix}$, then:

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} = \sqrt{(x_1' - y_1')^2 + (x_2' - y_2')^2}$$

$$= \sqrt{\left(a_{11}^2 + a_{21}^2\right)(x_1 - y_1)^2 + 2\left(a_{11}a_{12} + a_{21}a_{22}\right)(x_1 - y_1)(x_2 - y_2) + \left(a_{12}^2 + a_{22}^2\right)(x_2 - y_2)^2}$$