Math 487
Exam 2 - Practice Problems

1. Short Answer/Essay Questions:
   (a) Explain the difference between an axiomatic system and a model.
   (b) Which geometric model best describes the universe we live in?
   (c) Give an example of an SMSG axiom that is *not* independent of the other axioms.
   (d) Give an example of an SMSG axiom that *is* independent of the other axioms. Provide justification for your answer.
   (e) How do we know that the Euclidean Parallel Postulate is Independent of the other axioms?
   (f) Give the definition of an equivalence relation.
   (g) Give the definition of an angle bisector.

2. For each of the following, provide an example of a model where the statement is true and an example of a model where the statement is false.
   (a) There is a unique line between any pair of points.
   (b) The ruler postulate holds.
   (c) The SAS postulate holds.
   (d) the AAA congruence theorem is true.
   (e) The Plane Separation Postulate holds.
   (f) Rectangles exist.
   (g) Lines have infinite length.
   (h) the sum of the measures of the angles in any triangle is 180

3. For each statement, determine whether the statement is true or false. Then briefly justify your answer. Assume each statement is being made about a neutral geometry (i.e. every postulates except the Euclidean parallel postulate holds).
   (a) Parallel lines exist.
   (b) If a line ℓ is perpendicular with distinct lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$, then the lines $\overleftrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.
   (c) Given and 3 distinct collinear points, exactly one is between the other two.
   (d) If $m(\angle BAC) = m(\angle BAD) + m(\angle DAC)$, then $D \in int(\angle BAC)$.

4. Determine whether or not the Missing strip plane satisfies the Euclidean Parallel Postulate.

5. Prove that a line segment has a unique midpoint.

6. Prove that triangle congruence is an equivalence relation.

7. Prove that the Missing Strip Place does not satisfy Pasch's Postulate.

8. State and prove the AAS congruence theorem.

9. Prove that there are at least two lines that are parallel to each other.

10. Prove that the diagonals of a Saccheri quadrilateral are congruent.

11. Use the previous results along with the Theorem 2.18 and Euclid's 5th Postulate to prove that rectangles exist in a Euclidean geometry.

12. Prove each of the following Euclidean Propositions:

    (a) If A and D are points on the same side of a line $\overleftrightarrow{BC}$ and the line $\overleftrightarrow{BA}$ is parallel to the line $\overleftrightarrow{CD}$ then $m(\angle ABC) + m(\angle BCD) = 180$

    (b) Every distinct pair of parallel lines have a common perpendicular.

    (c) If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.

    (d) The sum of the measures of any triangle is 180.

13. Provide the justification for each step in the following 2-column proof (you may assume all SMSG axioms except the Parallel Postulate).

    Theorem (Exterior Angle Theorem): Any exterior angle of a triangle $\triangle ABC$ is greater than either of its remote interior angles.

    Proof: $\triangle ABC$, let D be a point such that $A - C - D$ (i.e. $\angle BCD$ is an exterior angle of $\triangle ABC$).

1. Let M be the midpoint of segment $\overline{BC}$.	
2. Then $B - M - C$ and $\overline{BM} \cong \overline{MC}$.	
3. There is a point E on ray $\overrightarrow{AM}$ such that $A - M - E$ and $ME = MA$.	
4. $\overline{ME} \cong \overline{MA}$	
5. $\angle AMB$ and $\angle EMC$ are vertical angles.	
6. $\angle AMB \cong \angle EMC$	
7. $\triangle AMB \cong \triangle EMC$	
8. $\angle ABC = \angle ABM \cong \angle ECM = \angle ECB$	
9. $m(\angle ABC) = m(\angle ECB)$	
10. E and D are on the same side of line $\overleftrightarrow{BC}$	
11. B, M and E are on the same side of line $\overleftrightarrow{CD}$	
12. $E \in int(\angle BCD)$	
13. $m(\angle BCE) + m(\angle ECD) = m(\angle BCD)$	
14. $m(\angle BCD) > m(\angle BCE) = m(\angle ABC)$	

The proof of the case for the other remote interior angle is similar $\square$.