Math 487
Exam 3 Review Sheet
Transformational Geometry

Definitions:

- Know the definitions of: a mapping, domain, co-domain, range, one-to-one, onto, a transformation, and a transformation of a plane.

Groups:

- Know the definition of a group.
- Be able to determine whether or not a given binary operation on a set is a group.
- Know (and be able to prove) which sets of transformations (all, affine trans., isometries, direct isometries, translations, rotations about a single point) form a group under function composition.
- Be able to investigate the group of symmetries of a given set points in the Euclidean plane.

The Analytic Model of $\mathbb{E}$:

- Know how to find the homogeneous coordinates of a line in this model.
- Know how to find the homogeneous coordinates of a point in this model.
- Know how to use matrix operations to determine whether or not a point is on a given line.
- Know how to use the determinant of a matrix to determine whether or not a set of three distinct points are collinear.
- Know how to use the determinant of a matrix to determine whether or not a pair of distinct lines are parallel.
- Be able to find the line containing a given pair of points and be able to find the point of intersection of a pair of non-parallel lines.
- Know how to compute distances and angles in this model.

Affine Transformations and Isometries:

- Know the definition and standard matrix representation of an affine transformation.
- Know that affine transformations preserve collinearity.
- Know the definition of and the standard matrix representations of direct and indirect isometries.
- Know that the determinant of a direct isometry is 1 and the determinant of an indirect isometry is -1.
- Understand the impact of composition (matrix multiplication) on pairs of direct and/or indirect isometries.
- Know that direct isometries preserve the measure of the angles between a pair of intersecting lines while indirect isometries reverse the measure of these angles.
- Know and be able to either prove or apply the fact that betweenness, collinearity, lines, segments, rays, circles, congruent triangles, and angle measure are all preserved under isometry.
- Know and be able to apply the fact that an isometry is completely determined by its action on three non-collinear points.
- Be able to determine whether or not a given function is: a transformation, an isometry.

Classifying Isometries:

- Know the definition of a translation T_{PQ}. Also know the general matrix form for a translation in $\mathbb{E}$ and be able to find the matrix representation of a specific translation. Understand the invariant sets of a translation.
- Know the definition of a rotation $R_{c,\theta}$. Also know the general matrix form for a rotation in $\mathbb{E}$ and be able to find the matrix representation of a specific rotation. Understand the invariant sets of a rotation.
- Know the definition of a reflection in a line ℓ R_ℓ. Also know the general matrix form for a reflection in $\mathbb{E}$ and be able to find the matrix representation of a specific reflection. Understand the invariant sets of a reflection.
- Know the definition of a glide reflection and the fact that any glide reflection can be represented as the composition of a reflection and a translation.
- Given a 3×3 matrix be able to identify whether or not it is a transformation, an affine transformation, an isometry, and be able to classify its type in the case where it is an isometry.
- Given a plane figure, be able to draw its image under a given isometry.
- Know the definition of the symmetries of a set of points and be able to investigate which isometries are in the group of symmetries of a given set of points.