Math 487 Properties Resulting from Different Parallel Postulates

Properties of Euclidean, Hyperbolic, and Elliptic Geometries:

| Euclidean | Hyperbolic | Elliptic |
|---|---|--|
| Two distinct non-parallel lines inter- sect in one point. | Two distinct non-parallel lines inter- sect in one point. | Two distinct non-parallel lines inter- sect in <i>at least</i> one point. |
| The sum of the measures of the angles of a triangle is 180. | The sum of the measures of the angles of a triangle is less than 180. | The sum of the measures of the angles of a triangle is more than 180. |
| Similar triangles that are not congruent exist. | Similar triangles are congruent. | Similar triangles are congruent. |
| Rectangles exist. | No quadrilateral is a rectangle. | No quadrilateral is a rectangle. |
| A line does not have finite length and is unbounded. | A line does not have finite length. | A line has finite length and is un- bounded. |
| Two parallel lines are equidistant. | No two parallel lines are equidistant. | Parallel lines do not exist. |
| The summit angles of a Saccheri quadrilateral are right angles. | The summit angles of a Saccheri quadrilateral are acute angles. | The summit angles of a Saccheri quadrilateral are obtuse angles. |
| Two distinct lines do not enclose a fi- nite area. | Two distinct lines do not enclose a fi- nite area. | Two distinct lines enclose a finite area. |
| The area of a triangle is equal to half the product of the base and height. | The area of a triangle is proportional to its "defect". | The area of a triangle is proportional to its "excess". |
| A unique line perpendicular to a given line through a point not on the line. | A unique line perpendicular to a given line through a point not on the line. | All lines perpendicular to a given line intersect in a least one point. |
| A line separates a plane. | A line separates a plane. | Lines may or may not separate a plane. |