MDEV 127 - Intermediate Algebra
Handout: Equations

A. Definitions

• An **equation** is a statement that two algebraic expressions are equal. Equations can involve one or more variables, constants, and mathematical operations.

• An equation **in x** is one that involves *only one variable*, namely, x.

Example: $x^3 + \sqrt{x} = 14x$

• A **root** or **solution** to an equation is a value that, when substituted for x into the equation, makes the equation true.

Example: $x = 0$ is a solution to the equation $x^3 + \sqrt{x} = 14x$ [Check this!]
Similarly, $x = 1$ is *not* a solution to this equation [Check this too!]

• Two equations that have the same **solution set** are called **equivalent**.

• To **solve** an equation is to find its solution set (i.e. *all* of its solutions).

• There are three main types of equations:

1. **Identities** are *always* true (i.e. every number is in the solution set).
2. **Conditional equations** are *sometimes* true.
3. **Inconsistent equations** are *never* true (i.e. the solution set is $\emptyset$)

Examples:

1. $3x - x = x + x$ is an identity equation.
2. $x^3 + \sqrt{x} = 14x$ is a conditional equation.
3. $x + 3 = x + 2$ is an inconsistent equation (or a contradiction).

Notes:
• To solve an equation, we generally apply mathematical operations to the equation that result in an equivalent equation that is easier to understand so that we can "read off" the solution set.

• Sometimes we will use mathematical operations that are useful in understanding the solution set of an equation but that *do not* result in an equivalent equation [i.e. they may introduce *extraneous solutions*]. We need to be especially careful to check our answers in situations where these methods are used.

Warning: The methods we use to solve equations are significantly different from those that we use to simplify algebraic expressions. That is due to the fact that equations have "two sides" while expressions only have "one side". It is important that you think about what you are doing and that you do not "mix and match" your methods.

B. Linear Equations

• A **linear equation in one variable** is any equation that can be put into the form $ax + b = 0$

• To solve linear equations, we usually *isolate the variable* (i.e. get it by itself on one side of the equation).

• Two operations that are useful in solving equations are:
– Adding/Subtracting a term to/from both sides of an equation.
– Multiplying/Dividing both sides of an equation by a **non-zero** constant.

Example:

$$\begin{array}{rcl} 10x + 3 & = & 0 \\ -3 & & -3 \\ 10x & = & -3 \\ \div 10 & & \div 10 \\ x & = & -\frac{3}{10} \end{array}$$

Note: As we move forward, we will look at more complicated equations and we will use these and other techniques in order to find their solution sets.

C. Solving Equations by Factoring

• To solve equations by factoring, we begin by expanding all of the expressions involved in the equation.

• Then, we move all the terms to one side, leaving zero on the other side of the equation.

• Next, we factor the expression on the left hand side completely.

• Finally, we use the zero factor property to split up the factors into individual smaller equations.

The Zero Factor Property: Given two real numbers r and s such that $r \cdot s = 0$, either $r = 0$ or $s = 0$ (or both).

Warning: Notice that this is a special property of zero. It does not work for any other real number. For example, if $r \cdot s = 1$, we cannot conclude that either r or s are 1 (we could have $r = 4$ and $s = \frac{1}{4}$, or infinitely many other non-zero pairs that multiply to give 1). For this reason, we can only use this method if we start with zero on one side of the equation!

Example:

$x(6x^2 - x - 2) = 0 \qquad (6 \cdot -2 = -12, so - 4 + 3 = -1)$

$x(6x^2 - 4x + 3x - 2) = 0$

$x\left[2x(3x - 2) + (3x - 2)\right] = 0$

$x(3x - 2)(2x + 1) = 0$

Using the zero factor Property, either:

$x = 0 \qquad 3x - 2 = 0 \qquad 2x + 1 = 0$

$x = 0 \qquad 3x = 2 \qquad 2x = -1$

$x = 0 \qquad x = \frac{2}{3} \qquad x = -\frac{1}{2}$