MDEV 127 - Intermediate Algebra
Handout: Properties of Exponents

A. Exponents

Definition: $a^n = a \cdot a \cdot a \cdot a \cdot \ldots \cdot a$ (a multiplied by itself n times) $\quad [x^5 = x \cdot x \cdot x \cdot x \cdot x]$

Properties of Exponents:

Property	**Example 1:**	**Example 2:**
$a^0 = 1$	$5^0 = 1$	$\left(\frac{4}{11}\right)^0 = 1$
$a^1 = a$	$5^1 = 5$	$\left(\frac{4}{11}\right)^1 = \frac{4}{11}$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{7}\right)^2 = \left(\frac{2^2}{7^2}\right) = \frac{4}{49}$	$\left(\frac{3}{5}\right)^3 = \left(\frac{3^3}{5^3}\right) = \frac{27}{125}$
$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^5 = x^{3+5} = x^8$	$\left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{2+3} = \left(\frac{1}{2}\right)^5 = \left(\frac{1^5}{2^5}\right) = \frac{1}{32}$
$(a^m)^n = a^{mn}$	$(x^3)^5 = x^{3\cdot 5} = x^{15}$	$\left[\left(\frac{1}{2}\right)^2\right]^3 = \left(\frac{1}{2}\right)^{2\cdot 3} = \left(\frac{1}{2}\right)^6 = \frac{1^6}{2^6} = \frac{1}{64}$
$(ab)^n = a^n b^n$	$(xy)^3 = x^3 y^3$	$(5x^2)^3 = 5^3 \cdot (x^2)^3 = 125x^{2\cdot 3} = 125x^6$
$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	$(x^3)^{-2} = \frac{1}{(x^3)^2} = \frac{1}{x^{3\cdot 2}} = \frac{1}{x^6}$
$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$	$\frac{x^5}{x^3} = x^{5-3} = x^2 = \frac{1}{x^{3-5}} = \frac{1}{x^{-2}}$	$\frac{3^5}{3^8} = 3^{5-8} = 3^{-3} = \frac{1}{3^{8-5}} = \frac{1}{3^3} = \frac{1}{27}$
$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$	$\frac{x^{-3}}{y^{-4}} = \frac{y^4}{x^3}$	$\frac{(2z)^{-4}}{(3p)^{-3}} = \frac{(3p)^3}{(2z)^4} = \frac{3^3p^3}{2^4z^4} = \frac{27p^3}{16z^4}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$	$\left(\frac{4}{11}\right)^{-2} = \left(\frac{11}{4}\right)^2 = \frac{121}{16}$	$\left(\frac{4s}{5p^2}\right)^{-2} = \left(\frac{5p^2}{4s}\right)^2 = \frac{5^2(p^2)^2}{4^2s^2} = \frac{25p^4}{16s^2}$

A More Complicated Example: Use the properties of exponents to simplify the expression: $\left(\dfrac{x^2y^{12}}{25x^3z^4}\right)^{-2}$

$$\left(\frac{x^2y^{12}}{25x^3z^4}\right)^{-2} = \left(\frac{25x^3z^4}{x^2y^{12}}\right)^2 = \left(\frac{25xz^4}{y^{12}}\right)^2 = \frac{25^2 \cdot x^2 \cdot (z^4)^2}{(y^{12})^2} = \frac{625x^2z^8}{y^{24}}$$

B. Scientific Notation:

If you have ever taken a Chemistry class, you may have encountered the following numbers:

There are approximately 602, 214, 179, 300, 000, 000, 000, 000 molecules in 1 mole of any substance (this quantity is called Avogadro's number).

The mass of an electron is approximately 0.0000000000000000000000000000009109382 grams.

You probably notice from these two examples that it is inconvenient to write very large or very small numbers using decimal notation. A more manageable way to write these numbers is to use scientific notation.

Definition: To express a number in scientific notation, we write it in the form $a \times 10^k$, where a is a number between 1 and 10, and k is an integer.

The number a is the first non-zero digit of the original number, followed by a decimal point, followed by all remaining digits down to the last nonzero digit in the original number.

The number k counts the number of places that the decimal point is moved to get from the original number to the number a. If the decimal is moved to the left, then k is a positive integer. If the decimal is moved to the right, then k is a negative integer.

Examples:

1. Write the number 27, 243.12 in scientific notation.
   $27,243.12 = 2.724312 \times 10^4$ (The decimal point was moved four places to the left).
2. Write the number 0.007239 in scientific notation.
   $0.007239 = 7.239 \times 10^{-3}$ (The decimal point was moved three places to the right).