MDEV 127 - Intermediate Algebra
Factoring

Factoring Methods

A. Greatest Common Factors

In this factoring method, you look at the expression to be factored and factor out the greatest common factor shared by *all* the terms in the expression (if there is one other than 1). This method should be employed before any other factoring method.

Example: Given the expression $15x^4y - 6x^3y^2 + 21x^2y^3$, the greatest common factor is: $3x^2y$

The result of factoring out this common factor is: $3x^2y\,(5x^2 - 2xy + 7y^2)$

B. Factoring by Grouping

This factoring method is used in expressions with an *even number of terms* (four or more).

To use this technique, follow these steps:

1. Group the terms together in two equal halves. Include in each half the terms that seem to have the most in common with each other.
2. Find and factor out the greatest common factor in each half separately.
3. After factoring out the greatest common factor in each half, look to see if the remaining grouped terms are the same [if they differ only by a minus sign, factor that out of one half].
4. Complete this method by grouping together the greatest common factors into a binomial term.

Example: Given the expression $x^3 - 4xy + 5x^2 - 20y$:

1. Group the terms together in two equal halves: $x^3 + 5x^2$ and $-4xy - 20y$
2. Find and factor out the greatest common factor in each half separately: $x^2(x + 5)$ and $-4y(x + 5)$
3. Look to see if the remaining grouped terms are the same: both have the term $(x + 5)$
4. Complete this method by grouping together the greatest common factors into a binomial term: $(x^2 - 4y)(x + 5)$

C. Factoring Trinomials (the "ac split")

This factoring method is used to factor quadratic expressions of the form: $ax^2 + bx + c$ or $au^2 + buv + cv^2$.

Follow these steps:

1. Write down all of the possible ways of factoring the product ac
2. Look for a combination that adds up to the b term
3. Split the b term into the sum of two terms that you found.
4. Complete this method by using factoring by grouping.

Example: Given the expression $6x^2 - 7x - 20$:

1. Write down all of the possible ways of factoring ac

   ac: $6 \cdot (-20) = -120$.

   This can be factored as: $1 \cdot (-120)$, $-1 \cdot 120$, $2 \cdot -60$, $-2 \cdot 60$, $3 \cdot -40$, $-3 \cdot 40$

   $4 \cdot -30$, $-4 \cdot 30$, $6 \cdot -20$, $-6 \cdot 20$, $8 \cdot -15$, $-8 \cdot 15$, $10 \cdot -12$, $-10 \cdot 12$

2. Look for a combination that adds up to the b term

   We first check: $1 - 120 = -119 \neq -7$ or $-1 + 120 = 119 \neq -7$

   Then we check $2 - 60 = -58 \neq -7$ or $-2 + 60 = 58 \neq -7$

   Continuing in this way, we eventually get to:

   $8 - 15 = -7$

3. Split the b term into the sum of two terms that you found.

   Since the combination we found was 8 and -15, we write $6x^2 + 8x - 15x - 20$

4. Complete this method by using factoring by grouping.

   Grouping, we get $6x^2 + 8x \quad -15x - 20$

   $= 2x(3x + 4) - 5(3x + 4)$

   Then the factorization is: $(3x + 4)(2x - 5)$

D. Special Factoring Formulas [You should *memorize* all of these factoring formulas]

1. Difference of Squares: $u^2 - v^2 = (u + v)(u - v)$

Example: $36x^2 - y^2 = (6x + y)(6x - y)$

2. Perfect Square: $u^2 + 2uv + v^2 = (u + v)^2$

Example: $4x^2 + 12x + 9 = (2x + 3)^2$

3. Sum of Cubes: $u^3 + v^3 = (u + v)(u^2 - uv + v^2)$

Example: $8x^3 + y^2 = (2x + y)(4x^2 - 2xy + y^2)$

4. Difference of Cubes: $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$

Example: $27x^3 - 8y^2 = (3x - 2y)(9x^2 + 6xy + 4y^2)$

5. (Non Rule) Sum of Squares: $u^2 + v^2$ Does Not Factor.