MDEV 127
Functions

Definition 1:
• A **function** is a correspondence or rule that assigns to each element in one set, called the domain D, *exactly one* element from a second set, called the range R.
• Alternatively, we can think of a **function** as a set of ordered pairs in which no two different ordered pairs have the same first coordinate. The domain D is the set of all *first coordinates*. The range R is the set of all *second coordinates*.

Definition 2:
• A **relation** is a correspondence or rule that assigns to each element in one set, called the domain D, *one or more* elements from a second set, called the range R.
• Alternatively, we can think of a **relation** as any set of ordered pairs. The domain D is the set of all *first coordinates*. The range R is the set of all *second coordinates*.

Examples:

1. Which of the following rules are functions?

Domain	Rule: add 3	Range
0	$0+3$	3
2	$2+3$	5
4	$4+3$	7
x	$x+3$	$x+3$

Domain	Rule: mult. by 2	Range
0	$0 \cdot 2$	0
2	$2 \cdot 2$	4
4	$4 \cdot 2$	8
x	$x \cdot 2$	$2x$

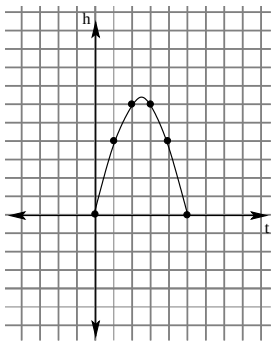
Domain	Rule: mult. by 2 then add 3	Range
0	$(0 \cdot 2)+3$	3
2	$(2 \cdot 2)+3$	7
4	$(4 \cdot 2)+3$	11
x	$(x \cdot 2)+3$	$2x+3$

2. $\{(0,1),(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8)\}$
3. $\{(0,1),(1,2),(2,3),(3,4),(4,5),(5,6),(6,5),(5,4),(4,3),(3,2),(2,1),(1,0)\}$
4. $\{(Brian, green),(Anna, blue),(Robert, red),(Allan, black),(Louise, pink),(Fred, yellow),(Bradley, red)\}$

Graphing:
To draw the graph of a function or relation involving numbers, we plot the ordered pairs defined by the rule for the function in the Cartesian Plane.

Example: Suppose our function is given by the rule $h = 5t - t^2$, where h represents the height of an object t seconds after it has been thrown.

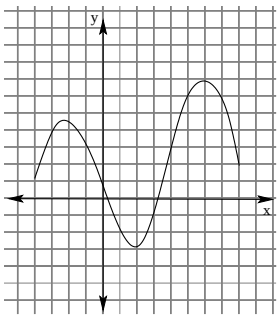
t	h
0	$5(0) - 0^2 = 0 - 0 = 0$
1	$5(1) - 1^2 = 5 - 1 = 4$
2	$5(2) - 2^2 = 10 - 4 = 6$
3	$5(3) - 3^2 = 15 - 9 = 6$
4	$5(4) - 4^2 = 20 - 16 = 4$
5	$5(5) - 5^2 = 25 - 25 = 0$

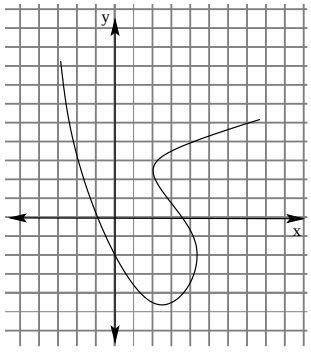


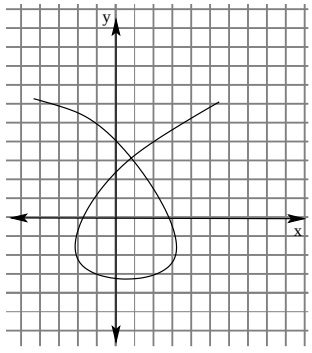
The Vertical Line Test:

A graph of points in the plane is the graph of a function if and only if every vertical line intersects the graph *at most* once.

Examples:

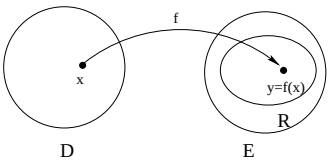


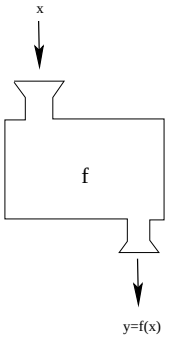




Which of these graphs are the graph of a function? For those that are, what are the domain and range?

Function Notation: Given a function f from a domain set D to a range set R, we will often use x to refer to elements from D and y to refer to elements from R. In this case, we will write out the rule for our function as: $y = f(x)$, read "y equals f of x".
We say that x the **argument** of f and we will call y the **value** of f at x.





• To evaluate a function, we input an x-value and find the corresponding value by applying the "rule" for the function to that input.

• Sometimes we may also want to work backwards. That is, for a given **output** y, we may want to find out which *input(s)* lead to the output y.

Examples:
Suppose $f(x) = \frac{x+1}{x-1}$. Then:

$f(2) = \frac{2+1}{2-1} = \frac{3}{1} = 3$

$f(-1) = \frac{-1+1}{-1-1} = \frac{0}{-2} = 0$

$f(2a-1) = \frac{(2a-1)+1}{2a-1-1} = \frac{2a}{2a-2} = \frac{2a}{2(a-1)} = \frac{2}{a-1}$

$f(3a-2b) = \frac{(3a-2b)+1}{(3a-2b)-1} = \frac{3a-2b+1}{3a-2b-1}$

If $f(x) = 2$, that what is x?

$\frac{x+1}{x-1} = 2$, so $x + 1 = 2(x-1) = 2x - 2$.

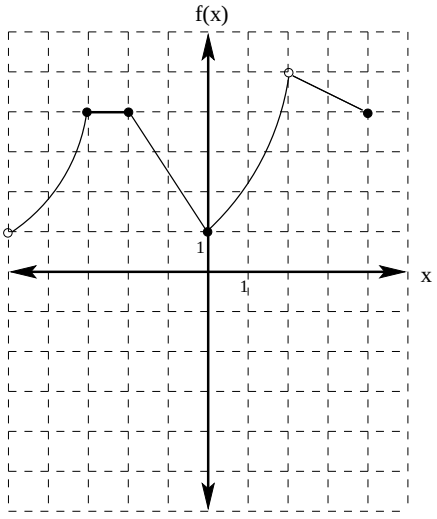
Then $x + 3 = 2x$, or $3 = x$. Check: $f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$.

The domain of f is ? _____

Example: Let $g(x) = x^2 + 2x - 3$ Find:

- $g(0)$
- $g(-1)$
- $g(3)$
- $g(2b)$

Example:



Find:
(a) $f(4)$

(b) x if $f(x) = 1$

(c) the domain of f

(d) the range of f