Math 127 - College Algebra
Handout: Properties of Exponents and Radicals

A. Exponents

Definition: $a^n = a \cdot a \cdot a \cdot a \cdot \ldots \cdot a$ (a multiplied by itself n times)

Properties:

1. $a^0 = 1$
2. $a^{-n} = \frac{1}{a^n}$
3. $a^m \cdot a^n = a^{m+n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^n = a^n b^n$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
7. $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$
8. $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
9. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

B. Radicals:

Definition: Suppose n is a positive integer and a is a real number. Then we define the **nth root of a**, denoted by $\sqrt[n]{a}$ as follows:
- If $a = 0$, then $\sqrt[n]{a} = 0$.
- If $a > 0$ then $\sqrt[n]{a}$ is the *positive* real number b such that $b^n = a$.
- If $a < 0$ and n is **odd**, then $\sqrt[n]{a}$ is the *negative* real number b such that $b^n = a$.
- If $a < 0$ and n is **even**, then $\sqrt[n]{a}$ is not a real number, since there is no real number b such that $b^n = a$.

Examples:
(a) $\sqrt[2]{9} = \sqrt{9} = 3$ since $3 \cdot 3 = 9$. (b) $\sqrt[3]{-8} = -2$ since $(-2) \cdot (-2) \cdot (-2) = -8$.

(c) $\sqrt{-16}$ is not a real number. (notice that $4 \cdot 4 = 16$, and $(-4) \cdot (-4) = 16$)

Properties:

1. $(\sqrt[n]{a})^n = a$ if $\sqrt[n]{a}$ is a real number.
2. $\sqrt[n]{a^n} = a$ if $a \geq 0$.
3. $\sqrt[n]{a^n} = a$ if $a < 0$ and n is odd.
4. $\sqrt[n]{a^n} = |a|$ if $a < 0$ and n is even.
5. $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ provided both exist.
6. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ provided both exist.
7. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ provided both exist.

Warning!!

(a) In general, $\sqrt{a^2 + b^2} \neq a + b$

(b) Also, in general, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

Exponents and Radicals:

1. $\sqrt[n]{a} = a^{\frac{1}{n}}$.
2. $\sqrt[n]{a^m} = a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$.