MDEV 127 - Intermediate Algebra
Handout: Properties of Real Numbers

A. Opposites and Reciprocals

Definitions:

• Two real numbers are **opposites** if they are the same distance from zero, but in opposite directions. Given a number r, its opposite is $-r$. The sum of a number and its opposite is always zero. $r + (-r) = 0$. For example, the opposite of $\frac{2}{3}$ is $-\frac{2}{3}$.

• Two real numbers are **reciprocals** if their product is 1. For example, the reciprocal of $\frac{2}{3}$ is the number $\frac{3}{2}$. Recall that when we multiply fractions, we take the product of the numerators and the product of the demoninators and then reduce: $\frac{2}{3} \cdot \frac{3}{2} = \frac{2 \cdot 3}{3 \cdot 2} = \frac{6}{6} = 1$.

B. Absolute Value:

Definition: The **absolute value** (or *magnitude*) of a real number r is the distance between r and zero. The absolute value of a real number r is written: $|r|$. Formally, we define absolute value as follows:

$$|r| = \begin{cases} r & \text{if } r \geq 0 \\ -r & \text{if } r < 0 \end{cases}$$

Examples:

(a) $\left|\frac{7}{4}\right| = \frac{7}{4}$ (b) $\left|-\frac{5}{12}\right| = \frac{5}{12}$ (c) $|\pi - 4| = 4 - \pi$

C. Real Number Properties:

Name:	**Symbolic Form:**	**Example:**
1. Commutative Property of Addition	$a + b = b + a$	$4 + 12 = 12 + 4$
2. Commutative Property of Multiplication	$a \cdot b = b \cdot a$	$127 \cdot 14 = 14 \cdot 127$
3. Associative Property of Addition	$a + (b + c) = (a + b) + c$	$4 + (x + 21) = (4 + x) + 21$
4. Associative Property of Multiplication	$a(bc) = (ab)c$	$4 \cdot (r \cdot 2) = (4 \cdot r) \cdot 2$
5. The Distributive Property	$a(b + c) = ab + ac$	$5(x + y) = 5x + 5y$
6. Additive Identity	$a + 0 = 0 + a = a$	$273 + 0 = 0 + 273 = 273$
7. Multiplicative Identity	$a \cdot 1 = 1 \cdot a = a$	$y \cdot 1 = 1 \cdot y = y$
8. Additive Inverse	$a + (-a) = 0$	$\pi + (-\pi) = 0$
9. Multiplicative Inverse	$a \cdot \frac{1}{a} = 1$	$\frac{2x}{3y} \cdot \frac{3y}{2x} = 1$

D. Adding Fractions:

• In order to add two fractions, we must modify the two fractions so that they have the same denominator. To do this, we use the prime factorization of the two denominators to find the **least common denominator** (or LCD).

• The LCD is found by multiplying together each prime factor that occurs. If a factor occurs more than once in a single factorization, we include it the maximum number of times that it occurs.

• Once we know the LCD, we multiply both the numerator and demoninator of each fraction by the appropriate number to get an equivalent fraction that has the LCD as its denominator.

• Then, now that both fractions have the same denominator, we add the fractions by adding the numerators (the denominator does not change).

• Finally, we reduce the resulting fraction to put it into lowest terms.

Examples:

(a) $\dfrac{5}{12} + \dfrac{4}{3}$ Since $12 = 3 \cdot 4$ and $3 = 3$, then the LCD is 3, so do not need to change the first fraction.

The second fraction becomes: $\dfrac{4}{3} \cdot \dfrac{4}{4} = \dfrac{16}{12}$. Then $\dfrac{5}{12} + \dfrac{4}{3} = \dfrac{5}{12} + \dfrac{16}{12} = \dfrac{21}{12} = \dfrac{7 \cdot 3}{4 \cdot 3} = \dfrac{7}{4}$

(b) $\dfrac{3}{24} + \dfrac{5}{16}$ Since $24 = 8 \cdot 3$ and $16 = 8 \cdot 2$, then the LCD is $8 \cdot 3 \cdot 2 = 48$.

The first fraction becomes $\dfrac{3}{24} \cdot \dfrac{2}{2} = \dfrac{6}{48}$. The second fraction becomes: $\dfrac{5}{16} \cdot \dfrac{3}{3} = \dfrac{15}{48}$.

Then $\dfrac{3}{24} + \dfrac{5}{16} = \dfrac{6}{48} + \dfrac{15}{48} = \dfrac{21}{48} = \dfrac{7 \cdot 3}{16 \cdot 3} = \dfrac{7}{16}$