

1. (4 points each) Calculate each of the following. Express your answers in simplest form.

$$(a) 3[6 + 2(4 - 2 \cdot 3)]$$

$$= 3[6 + 2(4 - 6)] = 3[6 + 2(-2)] \\ = 3[6 - 4] = 3(2) = 6$$

$$(b) 5^3 - 4^2 \div 2^2 - 3^4$$

$$= 125 - 16 \div 4 - 81 \\ = 125 - 4 - 81 = 121 - 81 = 40$$

$$(c) \frac{11}{12} - \frac{3}{16} = \frac{4}{4} \cdot \frac{11}{12} - \frac{3}{3} \cdot \frac{3}{16} = \frac{44}{48} - \frac{9}{48}$$

$$= \frac{35}{48} \text{ [Notice that } 35 = 7 \cdot 5 \text{ and } 48 = 3 \cdot 16 = 3 \cdot 2^4, \text{ so} \\ \text{this fraction is in reduced form.]}$$

$$(d) \frac{5}{8} \div \frac{25}{16}$$

$$= \frac{5}{8} \cdot \frac{16}{25} = \frac{5 \cdot 16}{8 \cdot 25} \\ = \frac{5 \cdot 8 \cdot 2}{8 \cdot 5 \cdot 5} = \frac{2}{5}$$

2. (3 points each) Given  $A = \{1, 3, 5, 7\}$ ,  $B = \{1, 2, 3, 4\}$ , and  $C = \{2, 4, 6, 8\}$ , find each of the following sets. Use proper set notation.

$$(a) A \cap B = \{1, 3\}$$

$$(b) A \cap (B \cup C) = A \cap \{1, 2, 3, 4, 6, 8\} = \{1, 3, 5, 7\} \cap \{1, 2, 3, 4, 6, 8\} = \{1, 3\}$$

$$(c) \{x \mid x \in A \text{ and } x \notin C\} = \{1, 3, 5, 7\}$$

3. (4 points each) Perform the following operations and simplify:

$$(a) 11x - 3(4x - 5y) + (4x - 7y) = 11x - 12x + 15y + 4x - 7y$$

$$= (11x - 12x + 4x) + (15y - 7y) = 3x + 8y$$

$$(b) \frac{3}{2}(x^2 + 4x) - \frac{5}{2}x$$

$$= \frac{3}{2}x^2 + \frac{12}{2}x - \frac{5}{2}x = \frac{3}{2}x^2 + \frac{7}{2}x$$

$$(c) 23 - 4(3 - 5)^2$$

$$= 23 - 4(-2)^2 = 23 - 4(4) = 23 - 16 = 7$$

$$(d) \text{ Subtract } 5x^2 - 7x + 2 \text{ from } 4x^2 + 3x - 7$$

$$= (4x^2 + 3x - 7) - (5x^2 - 7x + 2) \\ = 4x^2 + 3x - 7 - 5x^2 + 7x - 2 = -x^2 + 10x - 9$$

4. Use properties of exponents to simplify the following expression. Your answer should have no negative exponents. Assume all variables are non-zero.

(a) (2 points)  $-7^2 = -49$

(b) (3 points)  $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

(c) (5 points)  $\frac{12a^3b^7}{18a^4b^{-4}} = \frac{2 \cdot 6 \cdot a^3b^7b^4}{3 \cdot 6 \cdot a^4} = \frac{2b^{11}}{3a}$

(d) (5 points)  $\left(\frac{3x^4y^{-2}}{12x^2y^{-4}}\right)^3 = \left(\frac{3x^4y^4}{3 \cdot 4 \cdot x^2y^2}\right)^3 = \left(\frac{x^2y^2}{4}\right)^3 = \frac{x^6y^6}{64}$

5. (5 points each) Form the following products. Combine like terms. Box your answers.

(a)  $(7x - 5)(3x + 4) = 21x^2 + 28x - 15x - 20 = 21x^2 + 13x - 20$

(b)  $(2x - 5y)^2 = (2x - 5y)(2x - 5y) = 4x^2 - 10xy - 10xy + 25y^2 = 4x^2 - 20xy + 25y^2$

(c)  $(2x - 3y)(4x^2 + 6xy + 9y^2) = 8x^3 - 12x^2y + 12x^2y - 18xy^2 + 18xy^2 - 27y^3 = 8x^3 - 27y^3$

(d)  $(3x + 7)^2 - (3x - 7)^2$

$$= (9x^2 + 21x + 21x + 49) - (9x^2 - 21x - 21x + 49)$$

$$= 9x^2 + 21x + 21x + 49 - 9x^2 + 21x + 21x - 49 = 21x + 21x + 21x + 21x = 84x$$

6. (5 points each) Factor each of the following *completely*. Box your answers.

$$\begin{aligned} \text{(a)} \quad x^2 - x - 20 \quad ac = -20 = -5 \cdot 4 = x^2 - 5x + 4x - 20 \\ = x(x - 5) + 4(x - 5) \\ = (x - 5)(x + 4) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 6x^2 + 11x - 10 \quad ac = -60 = -4 \cdot 15 = 6x^2 + 15x - 4x - 10 \\ = 3x(2x + 5) - 2(2x + 5) \\ = (2x + 5)(3x - 2) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 16x^2 - 24xy + 9y^2 \quad a = 4x; b = 3y; -2ab = -2(4x)(3y) = -24xy \\ = (4x - 3y)^2 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 2x^3 + 3x^2 - 8x - 12 = x^2(2x + 3) - 4(2x + 3) \\ = (2x + 3)(x^2 - 4) \\ = (2x + 3)(x + 2)(x - 2) \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 27x^3 - 8 \quad u = 3x; v = 2 \quad \text{Pattern: } u^3 - v^3 \\ = (u - v)(u^2 + uv + v^2) \\ = (3x - 2)(9x^2 + 6x + 4) \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad x^4 - 16 \quad u = x^2; v = 4 \quad \text{Pattern: } u^2 - v^2 \\ = (x^2 + 4)(x^2 - 4) \\ = (x^2 + 4)(x - 2)(x + 2) \end{aligned}$$