MDEV 127 - Exam 2 (Version 1) **Name:**____________________
02/25/2011
You MUST show appropriate work to receive credit **Class Time:**____________________

1. (5 points each) Solve each of the following equations:

(a) $7x - 2 = 12$
$\quad +2 \quad +2$
$7x = 14$
$\frac{7x}{7} = \frac{14}{7}$
$x = 2$

(b) $4(x-2) + 5(3-2x) = 11$
$4x - 8 + 15 - 10x = 11$
$-6x + 7 = 11$
$\quad -7 \quad -7$
$-6x = 4$
$\frac{-6x}{-6} = \frac{4}{-6}$
$x = -\frac{2}{3}$

(c) $\dfrac{7}{10}x - \dfrac{4}{5} = \dfrac{5}{2}$
$10 \cdot \left(\dfrac{7}{10}x - \dfrac{4}{5}\right) = 10 \cdot \dfrac{5}{2}$
$\dfrac{70}{10}x - \dfrac{40}{5} = \dfrac{50}{2}$
$7x - 8 = 25$
$7x = 33$
$x = \frac{33}{7}$

(d) $-15x = 3x^2$
$0 = 3x^2 + 15x$
$0 = 3x(x+5)$
$0 = 3x$ or $0 = x + 5$
$x = 0$ or $x = -5$.

(e) $(x+6)(x-2) = 9$
$x^2 + 6x - 2x - 12 = 9$
$x^2 + 4x - 21 = 0$
$(x+7)(x-3) = 0$
$x + 7 = 0$ or $x - 3 = 0$
$x = -7$ or $x = 3$

(f) $|5 - 3x| - 2 = 7$
First isolate: $|5 - 3x| = 9$
$5 - 3x = 9$ or $5 - 3x = -9$
$-5 \qquad\qquad -5$
$-3x = 4$ or $-3x = -14$
$x = -\frac{4}{3}$ or $x = \frac{14}{3}$

(g) $|2x - 5| = |1 - 3x|$
$2x - 5 = 1 - 3x$ or $2x - 5 = -1 + 3x$
$\quad +5 \qquad\qquad +5$
$2x = 6 - 3x$ or $2x = 4 + 3x$
$5x = 6$ or $-x = 4$
$x = \frac{6}{5}$ or $x = -4$

2. (4 points) Solve $ax - 5 = bx + y$ for x.

$ax - bx = y + 5$
$x(a - b) = y + 5$
$x = \dfrac{y+5}{a-b}$

3. (4 points) 36 is 4% of what number?

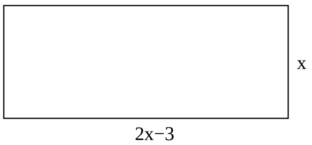
Let x be the unknown quantity. Then $36 = 0.04x$, or $36 = \frac{4}{100}x$
Therefore $3600 = 4x$, or $\frac{3600}{4} = x$
Hence $x = 900$.

The unknown quantity is 900.

4. (6 points each) Solve the following problems. You MUST define unknowns, set up and solve an equation, and clearly state your conclusion.

   (a) The length of a rectangular box is 3 cm. less than twice its width. The perimeter is 54 cm. Find the dimensions of this rectangle.

   Let x be the width of the rectangular box in centimeters. Then the length of the box is $2x - 3$ (see diagram).



   Recall that the perimeter of a rectangle is given by $P = 2w + 2\ell$. Then $54 = 2x + 2(2x - 3)$
   Therefore, $54 = 2x + 4x - 6$, or $60 = 6x$

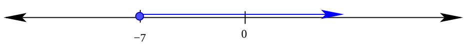
   Hence $x = \frac{60}{6} = 10$ centimeters.
   But then $\ell = 2x - 3 = 2(10) - 3 = 20 - 3 = 17$ centimeters.
   Thus the dimensions of the rectangular box are 10 cm. by 17 cm.

   (b) For a carnival event, adult tickets cost \$5 and kids under 12 pay \$4. If 50 tickets were sold for a total of \$215, how many of each type of ticket were sold?

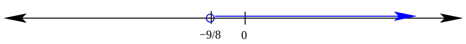
   Let x be the number of tickets sold to adults. Then there were $50 - x$ tickets sold to kids under 12.
   Since the revenue from ticket sales is found by taking the price times the quantity sold, total revenue is given by: $215 = 5x + 4(50 - x)$, or $215 = 5x + 200 - 4x$
   Then $15 = x$ and $50 - x = 50 - 15 = 35$
   Hence there were 15 tickets sold to adults and 35 tickets sold to kids under 12.
   But then $\ell = 2x - 3 = 2(10) - 3 = 20 - 3 = 17$ centimeters.
   Thus the dimensions of the rectangular box are 10 cm. by 17 cm.

5. (5 points each) Solve each of the following inequalities. Graph your answer on a number line. Then, write your answer in *interval notation.*

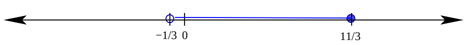
   (a) $21 \geq -3x$
   $\frac{21}{-3} \leq \frac{-3x}{-3}$
   $-7 \leq x$
   In interval notation, this is:
   $[-7, \infty)$



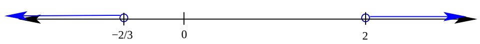
   (b) $.4x - .3(4x - 1) < 1.2$
   $10 \cdot [.4x - .3(4x - 1)] < 10 \cdot (1.2)$
   $4x - 3(4x - 1) < 12$
   $4x - 12x + 3 < 12$
   $-8x < 9$ $x > -\frac{9}{8}$
   In interval notation, this is:
   $\left(-\frac{9}{8}, \infty\right)$



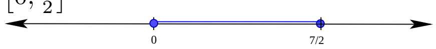
   (c) $-5 < 3x - 4 \leq 7$
   $+4 \qquad +4 \quad +4$
   $-1 < 3x < 11$
   $-\frac{1}{3} < x \leq \frac{11}{3}$
   In interval notation, this is:
   $\left(-\frac{1}{3}, \frac{11}{3}\right]$



   (d) $|3x - 2| > 4$
   $3x - 2 > 4$ or $3x - 2 < -4$
   $+2 \quad +2 \qquad +2 \quad +2$
   $3x > 6$ or $3x < -2$
   $X > 2$ or $x < -\frac{2}{3}$
   In interval notation, this is:
   $\left(-\infty, -\frac{2}{3}\right) \cup (2, \infty)$



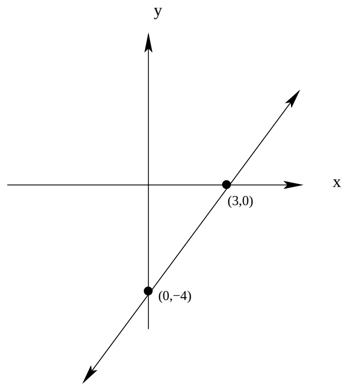
   (e) $|4x - 7| + 3 \leq 10$ $|4x - 7| \leq 7$
   $4x - 7 \leq 7$ and $4x - 7 \geq -7$
   $4x \leq 14$ and $4x \geq 0$
   $x \leq \frac{14}{4} = \frac{7}{2}$ and $x \geq 0$
   In interval notation, this is:
   $\left[0, \frac{7}{2}\right]$



6. (5 points each) Graph each of the following lines. Be sure to find and label the intercepts.

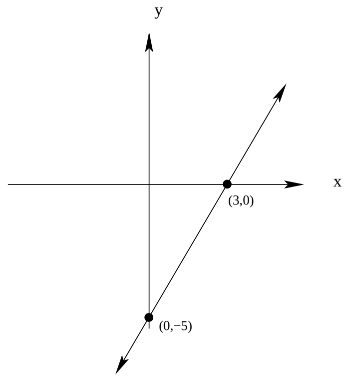
(a) $4x - 3y = 12$

x	y
0	-4
3	0



(b) $\frac{x}{3} - \frac{y}{5} = 1$

x	y
0	-5
3	0



7. (4 points each)

(a) Find the slope of the line through the points $(4, 1)$ and $(2, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{2 - 4} = \frac{-4}{-2} = 2$$

(b) Find c if the line through $(5, 4)$ and $(-1, c)$ has slope $m = -3$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{c - 4}{-1 - 5} = \frac{c - 4}{-6} = -3$$

Then $c - 4 = -3(-6) = 18$, so $c = 18 + 4$, or $c = 22$.

(c) Find the slope of L if it is parallel to the line containing the points $(3, -2)$ and $(4, 1)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-2)}{4 - 3} = \frac{3}{1} = 3$$

Since the line we are looking for is parallel to the given line, the slope of L must also be 3.

(d) Find the slope of L if it is perpendicular to the line given by the equation $3x - 2y = 7$

We begin by putting this linear equation into slope/intercept form:

$-2y = -3x + 7$, so $\frac{-2y}{-2} = \frac{-3}{-2}x + \frac{7}{-2}$.

Then $y = \frac{3}{2}x - \frac{7}{2}$.

Since the line we are looking for is perpendicular to the given line, its slope must be the negative reciprocal of the slope of the given line.
Therefore, the slope of L is $-\frac{2}{3}$.