MDEV 127 - Exam 3 Solutions
03/25/2011
You MUST show appropriate work to receive credit

Name:____________________

Class Time:____________________

1. (4 points each) Find the equation of each line L. Put your answer in slope/intercept form.

   (a) $(3,-1)$ and $(5,2)$ are on L.

   First, we compute the slope if this line: $m = \frac{2-(-1)}{5-3} = \frac{3}{2}$

   Next, we use the point/slope formula using m and one of the two points to find an equation: $y - y_1 = m(x - x_1)$

   $y - 2 = \frac{3}{2}(x-5)$ or $y = \frac{3}{2}x - \frac{15}{2} + 2$ so $y = \frac{3}{2}x - \frac{15}{2} + \frac{4}{2}$.

   Hence $y = \frac{3}{2}x - \frac{11}{2}$

   (b) $(3,-1)$ is on L and L is parallel to $3y - 5x = 6$

   We begin by finding the slope of the related line: $3y = 5x + 6$, so $y = \frac{5}{3}x + \frac{6}{3}$ or $y = \frac{5}{3}x + 2$.

   Next, we recall that the since the line we are looking for is parallel to the line above, the slope of the line if also $\frac{5}{3}$. From here, we use the point/slope formula using m and the given point to find an equation: $y - y_1 = m(x - x_1)$

   $y - (-1) = \frac{5}{3}(x-3)$ or $y + 1 = \frac{5}{3}x - 5$ so $y = \frac{5}{3}x - 6$.

   (c) L has an x-intercept of 2 and a y-intercept of -5

   First notice that the coordinates of the intercept points are $(2,0)$ and $(0,-5)$.

   Next, compute the slope if this line: $m = \frac{0-(-5)}{2-0} = \frac{5}{2}$

   Then since we already know that the y-intercept occurs when $y = -5$, the line has equation: $y = \frac{5}{2}x - 5$.

2. (5 points each) Graph the solution sets to each of the following inequalities. Label appropriate points.

   (a) $2x - 3y \geq 6$

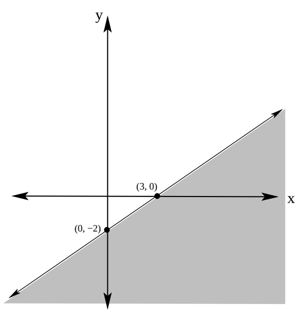
   We start by looking at the related line $2x - 3y = 6$ and plotting the intercepts.

x	y
0	-2
3	0

   Next, we look at a test point $(0,0)$.

   Notice that $2(0) - 3(0) = 0 \geq 6$ is FALSE

   From this, we graph the line and shade the region not containing the origin.



   (b) $x - 2 < 2y + 1$

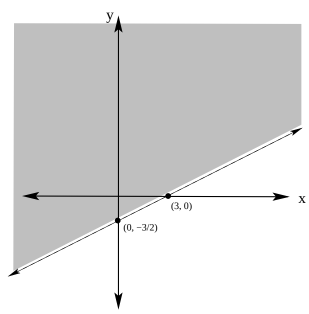
   We start by looking at the related line $x - 2 = 2y + 1$ and plotting the intercepts.

x	y
0	$-\frac{3}{2}$
3	0

   Next, we look at a test point $(0,0)$.

   Notice that $0 - 2 = -2 < 1 = 2(0) + 1$ is TRUE

   From this, we graph the line (using a dotted line) and shade the region containing the origin.



3. (2 points each) Suppose that a function f is given by the ordered pairs $\{(0,2),(1,-2),(2,5),(3,6),(4,2)\}$.

(a) Give the Domain of f in set notation.

$D = \{0,1,2,3,4\}$

(b) Give the Range of f in set notation.

$R = \{-2,2,5,6,\}$

(c) Find $f(2)$.

$f(2) = 5$ [Notice the pair $(2,5)$ indicates that then $x = 2$, $y = 5$.]

4. (3 points each) Assume $f(x) = 2x - 1$, $g(x) = x^2 + x$, and $h(x) = \dfrac{2}{x^2 - 1}$. Find:

(a) $g(7)$

$g(7) = 7^2 7 = 49 + 7 = 56.$

(b) $\dfrac{f}{g}(2)$

$f(2) = 2(2) - 1 = 4 - 1 = 3$
$g(2) = 2^2 + 2 = 4 + 2 = 6$
So $\dfrac{f}{g}(2) = \dfrac{3}{6} = \dfrac{1}{2}$

(c) $f(g(2))$

$g(2) = 2^2 + 2 = 6$

Then $f(g(2)) = f(6) = 2(6) - 1 = 12 - 1 = 11.$

(d) $(g \circ f)(x)$

$g(f(x)) = g(2x-1) = (2x-1)^2 + (2x-1)$

$= 4x^2 - 2x - 2x + 1 + 2x - 1 = 4x^2 - 2x.$

(e) $h(4)$

$h(4) = \frac{2}{16-1} = \frac{2}{15}$

(f) The domain of h in set notation.

Notice that $h(x) = \frac{2}{x^2-1} = \frac{2}{(x+1)(x-1)}$.

Then $h(x)$ is undefined when $x = \pm 1$.

Therefore, the domain of h is $D = \{x : x \neq 1, x \neq -1\}$

(g) The value of x for which $f(x) = 15$

If $f(x) = 15$, then $2x - 1 = 15$, so $2x = 16$, thus $x = 8$.

5. (6 points) Divide by a monomial. Box your answer. $\dfrac{12x^3y - 6x^4y^2 + 4x^2y}{2x^2y}$

$$\frac{12x^3y - 6x^4y^2 + 4x^2y}{2x^2y} = \frac{12x^3y}{2x^2y} - \frac{6x^4y^2}{2x^2y} + \frac{4x^2y}{2x^2y} = 6x - 3x^2y + 2$$

6. (7 points) Use long division to find the quotient $\dfrac{6x^2 + 11x - 4}{2x - 1}$. Box your answer, including any remainder.

$$\begin{array}{r|l} & 3x + 7 \\ \hline 2x - 1) & 6x^2 + 11x - 4 \\ & -\ 6x^2 + 3x \\ \hline & 14x - 4 \\ & -\ 14x + 7 \\ \hline & 3 \end{array}$$

Then $\dfrac{6x^2 + 11x - 4}{2x - 1} = 3x + 7 + \dfrac{3}{2x - 1}$.

7. (6 points) Simplify the following fraction as much as possible. Your answer should be completely reduced.

$$\frac{9x^2-4}{3x^2+7x-6}$$

$$\frac{9x^2-4}{3x^2+7x-6} = \frac{(3x+2)(3x-2)}{(3x-2)(x+3)} = \frac{3x+2}{x+3}$$

8. (8 points each) Perform the operations indicated and then simplify. Box your answers.

(a) $\frac{x-9}{x^2-9} \cdot \frac{x^2+4x+3}{x^2-8x-9}.$

$$= \frac{x-9}{(x+3)(x-3)} \cdot \frac{(x+3)(x+1)}{(x+1)(x-9)} = \frac{1}{x-3}.$$

(b) $\frac{x^2+7x+12}{x^2+8x+15} \div \frac{x+4}{x^2+3x-10}$

$$= \frac{x^2+7x+12}{x^2+8x+15} \cdot \frac{x^2+3x-10}{x+4} = \frac{(x+4)(x+3)}{(x+5)(x+3)} \cdot \frac{(x+5)(x-2)}{x+4}$$

$$= x-2$$

(c) $\frac{4x}{x^2+5x+6} - \frac{3x}{x^2+x-2}$

$$= \frac{4x}{(x+3)(x+2)} - \frac{3x}{(x+2)(x-1)} = \frac{4x(x-1)}{(x+3)(x+2)(x-1)} - \frac{3x(x+3)}{(x+3)(x+2)(x-1)}$$

$$= \frac{4x^2-4x-(3x+2+9x)}{(x+3)(x+2)(x-1)} = \frac{x^2-13x}{(x+3)(x+2)(x-1)} = \frac{x(x-13)}{(x+3)(x+2)(x-1)}.$$

(d) $\frac{x}{x+1} + \frac{3}{x-1} - \frac{6}{x^2-1}$

$$= \frac{x}{x+1} + \frac{3}{x-1} - \frac{6}{(x+1)(x-1)} = \frac{x(x-1)}{(x+1)(x-1)} + \frac{3(x+1)}{(x+1)(x-1)} - \frac{6}{(x+1)(x-1)}$$

$$= \frac{x^2-x)}{(x+1)(x-1)} + \frac{3x+3)}{(x+1)(x-1)} - \frac{6}{(x+1)(x-1)} = \frac{x^2+2x-3}{(x+1)(x-1)}$$

$$= \frac{(x+3)(x-1)}{(x+1)(x-1)} = \frac{x+3}{x+1}.$$