MDEV 127 - Exam 4 Solutions **Name:**_____
04/15/2011
You MUST show appropriate work to receive credit **Class Time:**_____

1. (6 points) Simplify the following expression as much as possible. Your answer should be completely reduced and should contain no complex fractions.

$$\frac{1+\frac{1}{x-1}}{1-\frac{1}{x+1}}$$

$$\frac{1+\frac{1}{x-1}}{1-\frac{1}{x+1}} \cdot \frac{(x-1)(x+1)}{(x-1)(x+1)} = \frac{(x-1)(x+1)+\frac{(x-1)(x+1)}{x-1}}{(x-1)(x+1)-\frac{(x-1)(x+1)}{x+1}} = \frac{(x-1)(x+1)+(x+1)}{(x-1)(x+1)-(x-1)}$$

$$= \frac{x^2=1+x+1}{x^2-1-x+1} = \frac{x^2+x}{x^2-x} = \frac{x(x+1)}{x(x-1)} = \frac{x+1}{x-1}$$

2. (6 points each) Solve each of the following equations.

(a) $\frac{a}{a-4} = \frac{1}{a+6}$

$$(a-4)(a+6) \cdot \frac{a}{a-4} = \frac{1}{a+6} \cdot (a-4)(a+6)$$

$a(a+6) = a-4$, or $a^2+6a = a-4$

Then $a^2+5a+4=0$, or $(a+4)(a+1)=0$

So $a=-4$ or $a=-1$

Check:
$\frac{-4}{-4-4} = \frac{-4}{-8} = \frac{1}{2}$
$\frac{1}{-4+6} = \frac{1}{2}$ ✓

$\frac{-1}{-1-4} = \frac{-1}{-5} = \frac{1}{5}$
$\frac{1}{-1+6} = \frac{1}{5}$ ✓

(b) $\frac{3}{x+6} - \frac{1}{x-2} = \frac{-6}{x^2+4x-12}$

$$(x+6)(x-2) \cdot \frac{3}{x+6} - \frac{1}{x-2} = \frac{-6}{(x+6)(x-2)} \cdot (x+6)(x-2)$$

$3(x-2)-1(x+6) = -6$, or $3x-6-x-6=-6$

Then $2x = 12-6 = 6$, so $x=3$

$\frac{3}{3+6} - \frac{1}{3-2} = \frac{3}{9} - \frac{1}{1} = \frac{1}{3} - 1 = -\frac{2}{3}$
$\frac{-6}{3^2+4(3)-12} = \frac{-6}{9+12-12} = \frac{-6}{9} = -\frac{2}{3}$ ✓

3. (6 points) Solve for y in the following equation: $x = \frac{5y-3}{3y+7}$

$$(3y+7)x = \frac{5y-3}{3y+7} \cdot (37+7)$$

$3xy+7x = 5y-3$, or $3xy-5y = -7x-3$

$y(3x-5) = -7x-3$, or $y = \frac{-7x-3}{3x-5}$

Note: If you divide by -1, you could also get $y = \frac{7x+3}{5-3x}$

4. (6 points each) Use algebra to solve each of the following. You must clearly define your variables and state your conclusion in a sentence.

   (a) One number is 4 times another. The sum of their reciprocals is $\frac{1}{4}$. Find the numbers.

   Let x be the smaller number and $y = 4x$ the larger number. Then $\dfrac{1}{x} + \dfrac{1}{4x} = \dfrac{1}{4}$

   Then $4x \cdot \dfrac{1}{x} + \dfrac{1}{4x} = \dfrac{1}{4} \cdot 4x$,
   or $4 + 1 = x$, so $x = 5$ and $4x = 20$.
   Check: $\dfrac{1}{5} + \dfrac{1}{20} = \dfrac{4}{20} + \dfrac{1}{20} = \dfrac{5}{20} = \dfrac{1}{4}$ ✓

   Therefore, the two numbers are 5 and 20.

   (b) A boat travels 24 miles upstream in the same time it takes to travel 30 miles downstream. The current is 2 mph. Find the speed of the boat in still water.

   Let x be the speed of the boat in still water, and recall that the rate equation is: $d = rt$, or $\frac{d}{r} = t$ where d is the distance traveled, r is the rate of travel, and t is the time spend traveling.

   Notice that if the speed of the boat in still water is x miles per hour, since the current is 2 mph, then the speed upstream is $x - 2$ and the speed downstream is $x + 2$.

	d	r	t
upstream	24	$x - 2$	t
downstream	30	$x + 2$	t

   Then we have $\dfrac{d_1}{r_1} = \dfrac{d_2}{r_2}$, or $\dfrac{24}{x-2} = \dfrac{30}{x+2}$

   Multiplying both sides by $(x-2)(x+2)$ gives:
   $24(x+2) = 30(x-2)$, or $24x + 48 = 30x - 60$.

   Then $60 + 48 = 30x - 24x$, or $108 = 6x$.
   Thus $x = \frac{108}{6} = 18$.

   Check: $\dfrac{24}{18-2} = \dfrac{24}{16} = \dfrac{3}{2}$ while $\frac{30}{18+2} = \frac{30}{20} = \frac{3}{2}$ ✓

   Therefore, the speed of the boat in still water is 18 miles per hour.

5. (3 points each) Simplify each of the following. Assume all variables represent nonnegative numbers.

   (a) $-\sqrt{64} = -\sqrt{8^2} = -8$.

   (b) $\left(\dfrac{9}{25}\right)^{\frac{3}{2}} = \left[\left(\dfrac{9}{25}\right)^{\frac{1}{2}}\right]^3 = \left[\dfrac{3}{5}\right]^3 = \dfrac{27}{125}$

   (c) $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}} = \left(\dfrac{27}{8}\right)^{\frac{2}{3}} = \left[\left(\dfrac{27}{8}\right)^{\frac{1}{3}}\right]^2 = \left[\dfrac{3}{2}\right]^2 = \dfrac{9}{4}$

   (d) $\sqrt{100x^4y^{12}z^6} = \sqrt{10^2x^4y^{12}z^6} = 10x^2y^6z^3$

   (e) $\sqrt[3]{24x^8y^{10}} = \sqrt[3]{2^3 \cdot 3 \cdot x^6x^2y^9y} = 2x^2y\sqrt[3]{3x^2y}$

   (f) $\sqrt[4]{16x^{10}y^{15}} = \sqrt[4]{2^4x^8x^2y^{12}y^3} = 2x^2y^3\sqrt[4]{x^2y^3}$

6. (3 points each) Simplify each of the following. **DO NOT** assume that the variables are positive.

(a) $\sqrt{36x^4y^2} = |6x^2y| = 6x^2|y|$

(b) $\sqrt{x^2-2xy+y^2} = \sqrt{(x-y)(x-y)} = \sqrt{(x-y)^2}$
$= |x-y|$

7. (3 points each) Use the properties of exponents to simplify each expression. Assume all variables are positive. Leave no negative exponents in your answers.

(a) $\left(x^{\frac{3}{5}}\right)^{\frac{5}{6}} \cdot x^{\frac{1}{2}}$

$= x^{\frac{3}{5}\cdot\frac{5}{6}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x$

(b) $\dfrac{\left(x^{-2}y^{\frac{2}{3}}\right)^3}{x^2y^{\frac{3}{2}}}$
$= \dfrac{x^{-6}y^2}{x^2y^{\frac{3}{2}}} = \dfrac{y^{\frac{4}{2}}}{x^6x^2y^{\frac{3}{2}}} = \dfrac{y^{\frac{1}{2}}}{x^8}$

8. (3 points each) In each of the following, perform the operations indicated and simplify.

(a) $\left(x^{\frac{1}{2}} - 3y^{\frac{1}{3}}\right)^2$

$= x - 3x^{\frac{1}{2}}y^{\frac{1}{3}} - 3x^{\frac{1}{2}}y^{\frac{1}{3}} + 9y^{\frac{2}{3}}$

$= x - 6x^{\frac{1}{2}}y^{\frac{1}{3}} + 9y^{\frac{2}{3}}$

(b) $\left(\sqrt{3x} + 2\sqrt{y}\right)\left(\sqrt{3x} - 2\sqrt{y}\right)$

$= 3x + 2\sqrt{3xy} - 2\sqrt{3xy} - 4y = 3x - 4y$

(c) $\sqrt{50} + \sqrt{18} - \sqrt{200}$

$= \sqrt{25\cdot 2} + \sqrt{9\cdot 2} - \sqrt{100\cdot 2}$

$= 5\sqrt{2} + 3\sqrt{2} - 10\sqrt{2} = -2\sqrt{2}$

(d) $\sqrt{\dfrac{7}{5}} - \sqrt{\dfrac{5}{7}}$

$= \dfrac{\sqrt{7}}{\sqrt{7}} \cdot \sqrt{\dfrac{7}{5}} - \sqrt{\dfrac{5}{7}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{49}}{\sqrt{35}} - \dfrac{\sqrt{25}}{\sqrt{35}}$

$= \dfrac{7-5}{\sqrt{35}} = \dfrac{2}{\sqrt{35}} \cdot \dfrac{\sqrt{35}}{\sqrt{35}} = \dfrac{2\sqrt{35}}{35}$

9. Rationalize all denominators and simplify. Assume all variables represent positive values.

(a) (3 points) $\dfrac{2}{\sqrt{6}}$

$= \dfrac{2}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$

(b) (4 points) $\dfrac{5}{\sqrt[3]{2y^2}}$

$= \dfrac{5}{\sqrt[3]{2y^2}} \cdot \dfrac{\sqrt[3]{4y}}{\sqrt[3]{4y}} = \dfrac{5\sqrt[3]{4y}}{\sqrt[3]{8y^3}} = \dfrac{5\sqrt[3]{4y}}{2y}$

(c) (5 points) $\dfrac{\sqrt{3}-\sqrt{x}}{\sqrt{x}+\sqrt{5}}$

$= \dfrac{\sqrt{3}-\sqrt{x}}{\sqrt{x}+\sqrt{5}} \cdot \dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}} = \dfrac{(\sqrt{3}-\sqrt{x})(\sqrt{x}-\sqrt{5})}{(\sqrt{x}+\sqrt{5})(\sqrt{x}-\sqrt{5})}$

$= \dfrac{\sqrt{3x} - x - \sqrt{15} + \sqrt{5x}}{x-5}$

10. (6 points each) Determine all solutions to each of the following equations.

(a) $\sqrt[3]{2x} + 4 = 3$

$\sqrt[3]{2x} = -1$, or $\left(\sqrt[3]{2x}\right)^3 = (-1)^3$

Then $2x = -1$, or $x = -\frac{1}{2}$

Check: $\sqrt[3]{2\left(-\frac{1}{2}\right)} + 4 = \sqrt[3]{-1} + 4 = -1 + 4 = 3$ ✓

(b) $x + 5 = \sqrt{2x + 13}$ $(x + 5)^2 = \left(\sqrt{2x + 13}\right)^2$

$x^2 + 10x + 25 = 2x + 13$, so $x^2 + 8x + 12 = 0$

Then $(x + 6)(x + 2) = 0$

Check:

$-6 + 5 = -1$, $\sqrt{2(-6) + 13} = \sqrt{1} = 1$

Does not check!

$-2 + 5 = 3$, $\sqrt{2(-2) + 13} = \sqrt{9} = 3$ ✓

Thus the only solution is $x = -2$.

(c) $\sqrt{x + 8} = 2 + \sqrt{x}$

$\left(\sqrt{x + 8}\right)^2 = \left(2 + \sqrt{x}\right)^2$

Then $x + 8 = 4 + 4\sqrt{x} + x$, or $4 = 4\sqrt{x}$

Thus $1 = \sqrt{x}$, so $(1)^2 = (\sqrt{x})^2$ or $1 = x$

Check:
$\sqrt{1 + 8} = \sqrt{9} = 3$. $2 + \sqrt{1} = 2 + 1 = 3$ ✓