

1. Consider the function

$$f(x) = \frac{2^x - 8}{\sqrt{x^2 + 1} - \sqrt{10}}.$$

- (a) Use the graph of f to approximate the left-hand limits, right-hand limits, and limits at -3 and at 3 . Make sure that the six limits are clearly stated in complete sentences.
- (b) Find the same six limits as above, but do so numerically via matrix tables. Make sure that the estimated six limits are clearly stated in complete sentences.
- (c) Find the same six limits as above, but do so symbolically rather than graphically or numerically. Further, determine the value of the function at -3 and 3 .
- (d) If any discrepancies occurred in or between (a), (b), or (c), identify them and explain why you think they occurred.
- (e) Does the function have any horizontal asymptotes? Justify.
2. Determine, if possible, the left-hand limit, right-hand limit, and limit for

$$g(x) = \begin{cases} \frac{\tan 4x}{3x} & \text{if } x < 0 \\ (1 + 4x)^{\frac{1}{3x}} & \text{if } x \geq 0 \end{cases}$$

at 0. Evaluate g at 0. Also, determine the limit of the function g as x grows without bound to the left and right. *Are your solutions consistent with the graph of the function?*

3. Evaluate each limit, if possible. Use the same variable as given. Note: You should be looking at your answers to make sure that they are reasonable. If a solution is not reasonable, make sure that you have typed in the function correctly.

(a) $\lim_{t \rightarrow \infty} \frac{4t - 3}{\sqrt{t^2 + 1}}$

(e) $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 + \sin \theta}{\cos^2 \theta}$

(b) $\lim_{t \rightarrow -\infty} \frac{4t - 3}{\sqrt{t^2 + 1}}$

(f) $\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt{x^2 + 1}} - \frac{1}{x} \right)$

(c) $\lim_{\varphi \rightarrow 0} (\cot \varphi)^{\tan \varphi}$

(g) $\lim_{t \rightarrow 0^-} \left(\frac{1}{t} - \csc t \right)$

(d) $\lim_{x \rightarrow 0^+} (1 + 3x)^{\csc x}$

(h) $\lim_{\theta \rightarrow \frac{\pi}{2}^-} (\sec^2 \theta - \tan^2 \theta)$