Conditional Probability and Expected Value

1. Assume A and B are events in S such that $P(A) = .6$, $P(B) = .5$, and $P(A \cap B) = .2$.
   Draw a probability Venn diagram and then answer the following:

   (a) $P(A|B) =$

   (b) $P(B|A) =$

   (c) $P(B|A') =$

   (d) $P(A \cup B) =$

   (e) Are the events A and B independent events? Explain.

2. Assume 3 red and 6 blue chips are in a bowl. Suppose two chips are drawn, one at a time, without replacement. Determine the probability that:

   (a) The first chip is red **and** the second chip is blue.

   (b) Both chips are red.

   (c) At least one chip is blue.

   (d) One chip is red and one chip is blue.

3. Suppose you pay $1 to play the following game: You toss 3 coins. If all three coins are the same (all heads or all tails), then you win $6 (your original $1 plus $5 more). Otherwise, you lose your $1. Calculate the expected value for this game. Is this game fair?

4. While walking in the forest, you encounter a leprechaun who has two bags filled with coins. Bag A has 3 silver coins and 7 gold coins. Bag B has 6 silver coins and 2 gold coins, but you do not know which is which. The leprechaun offers to let you randomly select one of the bags and then randomly select one coin from that bag. (Hint: a tree diagram may be helpful here)

   (a) Find the probability that the coin you choose is gold.

   (b) Find the probability that the coin you chose came from Bag B, given that it is gold.

   (c) If the gold coins are worth $20 while the silver coins are only worth $5, what is the expected value of this transaction?