Math 102
Translating and Negating Logical Statements

Translating Statements: One important skill that we will be working on is the ability to translate logical statements. We will need to be comfortable taking statements written in English and writing them in symbolic form. We will also need to be comfortable taking statements represented in symbolic form and writing them in plain English.

Examples: Choose variables and then write each statement in symbolic form.

- I was late for class this morning or I set my alarm clock last night.
- I set my alarm clock last night and I was not late for class this morning.
- If I was late for class this morning then I did not set my alarm clock last night.
- I was not late for class this morning if and only if I set my alarm clock last night.

To translate these statements into symbolic form, we first need to assign variables to the simple statements involved in these compound statements.
We define p: "I was late for class this morning". q: "I set my alarm clock last night"
Then the symbolic form for these compound statements are as follows.

- $p \vee q$
- $q \wedge \sim p$
- $p \rightarrow \sim q$
- $\sim p \longleftrightarrow q$

Examples: Given the variables: p : "I go to class every day.", q :"I do my homework.", and r : "I get a good grade in this course.", translate each symbolic statement into plain English.

- $p \wedge q$
- $q \vee \sim r$
- $r \rightarrow (q \wedge p)$
- $(\sim p \vee \sim q) \rightarrow \sim r$

Negating Statements: Another important skill is the ability to find the negation of a logical statements. We will need to be practice taking a statement and finding its completely simplified negation. We will not consider the negation to be completely simplified until the negatives have been moved as close to each simple statement as possible and all "double negatives" have been removed.

Examples: Find the completely simplified negation of each of the following statements.

- I like getting up early in the morning.
- I am taking math and history this semester.
- I will go to the grocery store or I will go to the movies this evening.
- I am sick today and I will not go to work.
- All cows eat grass.
- Some cows eat hay.
- At least one cow wears a bell.
- No cows bark.

Notes:

- We need to be careful when we negate "and" statements and when we negate "or" statements. To negate these correctly, it is useful to remember DeMorgan's Laws:

  $\sim (p \vee q) = \sim p \wedge \sim q$

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  That is, the negation of an "and" statement is an "or" statement. The negation of an "or" statement is an "and" statement. We will prove DeMorgan's Laws using truth tables later in the course.

- We also need to be careful when we negate statements involving quantifiers. The negation of a statement is its "logical opposite", that is, the statement that is true whenever the original statement is false and false whenever the original statement is true. Because of this, one should remember the following:

  The negation of a universal quantifier is an existential quantifier. ["all" becomes "some are not"]

  The negation of am existential quantifier is a universal quantifier. ["some" becomes "all are not"]