Math 102
Introduction to Logic

Reasoning Methods:

Inductive Reasoning is the process of drawing a general conclusion by observing a pattern based on specific instances. This conclusion is called a **hypothesis** or **conjecture**.

Examples: Use inductive reasoning to predict the next two terms in the following sequences.

- 1, 3, 5, 7, _____, _____
- 2, 3, 5, 7, 11, _____, _____
- 1, 4, 9, 16, 25, _____, _____
- 1, 1, 2, 3, 5, 8, _____, _____

Caution: It is difficult to know whether or not the conclusions we reach using inductive reasoning are correct. There may be more than one conclusion that fits the observations we made. Hypotheses and conjectures must be held tentatively until we find a way to validate them.

Deductive Reasoning is the process of using premises (accepted facts) and logical principles to arrive at a specific conclusion.

Examples: Use deductive reasoning to reach a conclusion in each of the following.

- All rabbits have long ears. Fluffy does not have long ears. Conclusion?
- Find all numbers that satisfy the equation $x^2 = 4$.

Caution: The validity of the conclusions that we reach using deductive reasoning depend on the validity of our underlying assumptions. Also, in many situations where deductive reasoning is used, the underlying premises are not always clearly stated.

Statements, Logical Connectives, and Quantifiers:

- A **statement** is a declarative sentence that is either true or false. Statements are usually represented by the lowercase letters $p, q, r, \ldots$.

Examples and Non-Examples: Determine which of the following are statements.

- I was late for class today.
- Gas prices were lower today than they were yesterday.
- Gas prices will be lower tomorrow than they were today.
- What time is it?
- Be careful.
- This statement is true.
- This statement is false.

More Definitions:

- A **simple statement** is a statement that contains a single idea.
- A **compound statement** is a statement that contains at least two separate ideas joined together by a logical operation or "connective".

Examples:

- The best things in life are free.
- I will go to class today and then I will go to work.

Logical Connectives (Logical Operations): We will use 5 logical connectives to build up compound statements from given simple statements. The following table gives the name, meaning, and symbol for each of the 5 main logical connectives.

Name:	**Usage/Meaning**	**Mathematical Symbol**
Negation	"not" (the logical opposite)	$\sim$
Conjunction	"and" (both)	$\wedge$
Disjunction	"or" (inclusive or)	$\vee$
Conditional	"if/then"	$\rightarrow$
Biconditional	"if and only if"	$\longleftrightarrow$

Note: In the English language, "or" is used two different ways:

- Given two options, an *inclusive or* is satisfied when one option, the other option, or both options occur.
- An *exclusive or* is satisfied when one option occurs or when the other option occurs, but **not** when *both* options occur.

Examples:

- The statement: "I will buy apple juice or orange juice at the store" is an example of an *inclusive or*.
- The statement: "I will dye my hair blue or I will leave it its natural color" is an example of an *exclusive or*.
- The statement: "You will get soup or salad with your dinner" is an example of an _____.
- The statement: "I will go fishing or I will mow the lawn this weekend" is an example of an _____.

When the symbol $\vee$ is used, it will **always** represent an *inclusive or*.

Quantifiers:

- A **universal quantifier** is a quantifier that refers to **all** of the objects in a given category. It can be expressed positively using "For All" or "Every", or negatively using "No" or "None".
- An **existential quantifier** is a quantifier that refers to **a portion** of the objects in a given category. It can be expressed using "There Exists", "Some", or "At Least One".

Examples:

- All cows eat grass.
- Some cows eat hay.
- At least one cow wears a bell.
- No cows bark.