Math 102
Normal Distribution Examples

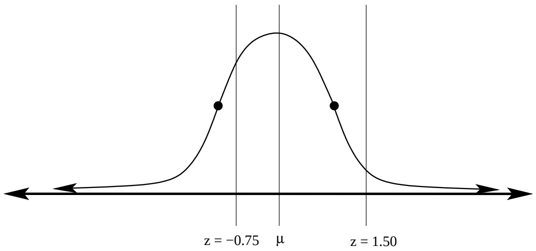
Although the 68%-95%-99.7% Rule is useful, often we want more detailed information about the percentage of the population that is under a portion of a normal distribution. One way to get more detailed information is to compute the z-score for a data value in normal distribution and then make use of the standard z-table.
The z-score of a raw data score x in a normal distribution with mean μ and standard deivation σ is found by computing:
$$z = \frac{x-\mu}{\sigma}.$$
The meaning of the z score is the number of standard deviations the data value is above or below the mean (above if the z-score is positive, below if the z-score is negative).

Examples:

(a) A z-score $z = 1.5$ means that that data value is 1.5 standard deviations above the mean.

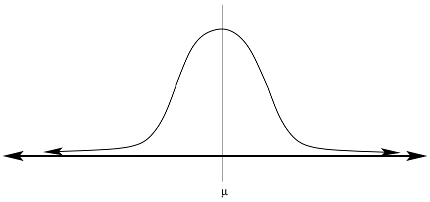
(b) A z-score $z = -.75$ means that that data value is 0.75 standard deviations below the mean.

If we look the resulting z-score up in the standard normal z-table, we find a decimal that represents the proportion of the population that is between the mean and the data value with the corresponding z-score. Notice that there are no negative z-values in the table. This is because we can use positive values and symmetry to find the proportions for negative z-scores.

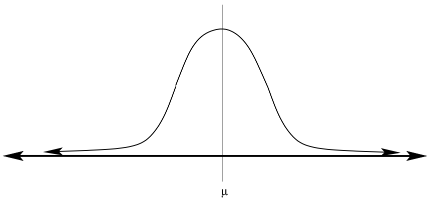


Example 1: Suppose that a given population is normal with mean $\mu = 20$ and standard deviation $\sigma = 5$.

(a) Find the z-score associated with the data values $x = 15$ and $x = 24$.

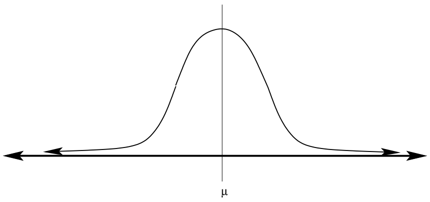


(b) Find the percentage of the population between $x = 15$ and $x = 24$



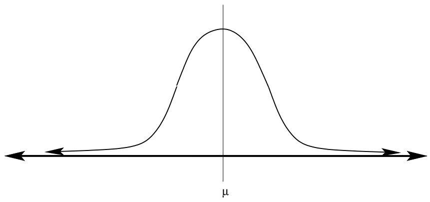
We can also work backwards from the z score to find the raw score associated with it. If we solve for x in the formula $z = \frac{x-\mu}{\sigma}$, we get $x = z\sigma + \mu$

(c) How big would x need to be in order for only 10% of the population to be higher than x?

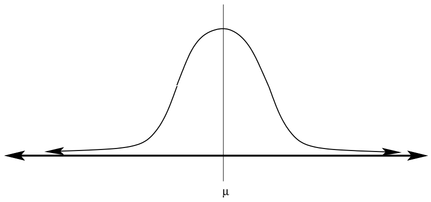


Example 2: Suppose that the number of slices of pizza consumed by students at MSUM on a weekly basis is normally distributed with mean $\mu = 5$ and standard devaition $\sigma = 1.5$.

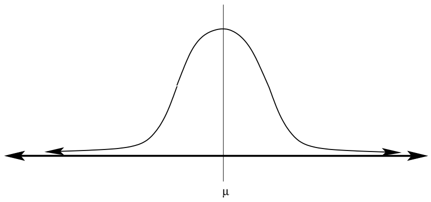
1. What percentage of MSUM students eat more than 8 pieces of pizza per week?



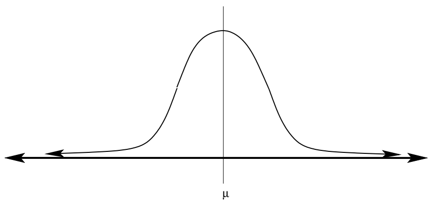
2. What percentage of MSUM students eat less than 4 pieces of pizza per week?



3. What percentage of MSUM students eat between 2 and 7 pieces of pizza per week?



4. How many pieces of pizza would a student have to eat in order to eat more pizza than 80% of the students at MSUM?



5. If there are 7,000 students currently enrolled at MSUM, how many of them eat more than 6 pieces of pizza per week?