Math 102
Problem Solving Handout

Problem Solving

Problem solving is often difficult because there is no single procedure that works all the time – each problem is slightly different. Also, problem solving requires practical knowledge about the specific situation. If you misunderstand either the problem or the underlying situation you will end up making mistakes or incorrect assumptions. One of our main goals this semester is to become better problem solvers. To that end, we will now discuss a framework for thinking about problem solving: a four stage procedure.

1. **Preparation:**
   - Learn the necessary underlying mathematical concepts.
   - Consider the terminology and notation used in the problem (what sort of problem is it? What is being asked?)
   - Draw a picture or diagram to represent the situation.
   - Rephrase the problem in your own words.
   - Write down specific examples of the conditions given in the problem.
2. **Thinking Time:**
   - Once you understand what the problem is asking, if you are stumped or stuck, set the problem aside for a while. Your subconscious mind will keep working on it.
   - Moving on to think about other things will help you stay relaxed, flexible, and creative rather than becoming tense, frustrated, and forced in your efforts to solve the problem.
3. **Insight:**
   - Once you have thought about a problem or returned to it enough times, you will often have a flash of insight: a new idea to try or a new perspective on how to approach solving the problem.
   - Once you have an idea for a new approach, jot it down immediately. When you have time, try it out and see if it leads to a solution. Keep trying until something works.
4. **Verification:**
   - Once you have a potential solution, check to see if it works.
   - Double check to make sure that all of the conditions related to the problem are satisfied by your potential solution. Also double check any computations involved in finding your solution.
   - If you find that your solution does not work, there may be only a simple mistake between you and a correct solution. Try to fix or modify your current attempt before scrapping it. Remember what you tried – it is likely that at least part of it will end up being useful.

Remember, problem solving is as much an art as it is a science!!
Next, we will look at 7 basic problem strategies: (it would be a good idea to memorize these!)

1. **Draw Pictures**
2. **Choose Good Names For Unknowns**
3. **Be Systematic**
4. **Look for Patterns**
5. **Try a Simpler Version of the Problem**
6. **Try Guided Guessing** (Guessing is OK)
7. **Convert a New Problem into an Older One**

Lastly, we will look at some basic mathematical principles that we will need to keep in mind when problem solving:

1. **The Always Principle:**

   Unlike many other areas, when we say a mathematical statement is true, we mean that it is true 100 percent of the time. We are not dealing with the uncertainty of statements that are "usually true" or "sometimes true".

2. **The Counterexample Principle:**

   Since a mathematical statement is true only when it is true 100 percent of the time, we can prove that it is **false** by finding a single example where it is not true. Such an example is called a *counterexample*.

   Of course, when we say a mathematical statement is false, this does not mean that it is never true – it only means that it is not always true. It might be true some of the time.

3. **The Order Principle:**

   In mathematics, **order usually matters**. In a multi-step mathematical process, if we carry the steps out in a different order, we often get a different result. For example, putting your socks on first and then your shoes is quite different from putting your shoes on first and then your socks.

4. **The Splitting Hairs Principle:**

   In mathematics, details matter. Two terms or symbols that look and sound similar may have mathematical meanings that are significantly different. For example, in English, we use the term equal and equivalent interchangeably, but in mathematics, these terms do not mean the same thing. For this reason, learning and remembering the precise meaning of mathematical terms is essential.

5. **The Analogies Principle:**

   Often the formal terminology used in mathematics has been drawn from words and concepts used in everyday life. This is not a coincidence. Associating a mathematical concept with its "real world" counterpart can help you remember both the formal (precise) and intuitive meanings of a mathematical concept.

6. **The Three Way Principle:**

   When approaching a mathematical concept, it often helps to use 3 complimentary approaches:

   • Verbal - make analogies, put the problem in your own words, compare the situation to things you may have seen in other areas of mathematics.

   • Graphical - draw a graph or a diagram.

   • Examples - Use specific examples to illustrate the situation.

   By combining one or more of these approaches, one can often get a better idea of how to think about and how to solve a given problem.