Math 102
Exam 1

Name:________________________________

1. (3 points each) For each of the following, state whether the situation is an example of inductive or deductive reasoning:

   (a) You notice that your houseplants seem to grow better if you water them in the morning rather than in the evening, so you decide to start watering them every morning right before you leave to go to school.

   Since you are using an observed pattern to arrive at a conclusion, this is an example of inductive reasoning.

   (b) After hearing a debate on the radio, you decide to construct a truth table in order to determine whether or not the logical argument given by one of the participants is valid.

   Since you are using established logical and mathematical principles to arrive at a conclusion, this is an example of deductive reasoning.

2. (3 points each) Determine whether or not each of the following are statements:

   (a) This box is full of fragile objects.

   This is a statement. Notice that it is a declarative sentence that is either true or false.

   (b) Handle it with care.

   This is not a statement. Notice that the sentence is a command and that that one cannot assign it a truth value.

3. (4 points each) Given p : roses are red, q : violets are blue, and r : sugar is sweet, translate the following statements into words:

   (a) $p \vee \sim r$

   Roses are red or sugar is not sweet.

   (b) $q \rightarrow (\sim r)$

   If violets are blue then sugar is not sweet.

4. (4 points each) Negate each of the following statements. Your final answer should be written in plain English with the negation moved as far to the right as possible.

   (a) Sometimes I am tired at the end of the workday.

   I am never tired at the end of the workday. (Recall that the negation of an existential quantifier is a universal quantifier).

   (b) I will buy a new coat or I will buy new mittens.

   I will not buy s new coat and I will not buy new mittens. (We are using De Morgan's Law here.)
   Also acceptable: I will neither buy a new coat nor will I buy new mittens.

5. (4 points each) For each of the following, decide whether the "or" used in the situation described is "inclusive" or "exclusive". Make sure to briefly explain your reasoning.

   (a) I will go for a jog after work or I will go shopping this evening.

   This is an example of an inclusive or. Notice that it is possible to do both activities.

   (b) I will dye my hair blue or I will leave it its natural color.

   This is an example of exclusive or. Notice that one cannot both dye one's hair blue and leave it its natural color.

6. (4 points each) Given the statements: p : I enjoy watching foreign films q : I go to the Fargo Theater.

   (a) Write the conditional statement relating p to q in words.

   If I enjoy watching foreign films than I go to the Fargo Theater.

   (b) Write the inverse statement in words.

   If I do not enjoy watching foreign films then I do not go to the Fargo Theater.

7. (8 points) According to De Morgan's Law, $\sim (p \vee q)$ is logically equivalent to $\sim p \wedge \sim q$. Use truth tables to prove that these two statements are logically equivalent.

p	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Since the last column in these two truth tables match, the statements are logically equivalent.

8. (7 points) Given that p is false, q is true, r is false, and s is true, what is truth value of the statement: $(p \vee \sim q) \rightarrow (\sim r \wedge s)$
[Hint: since we are only considering one truth value assignment, you do not need to build the entire truth table.]

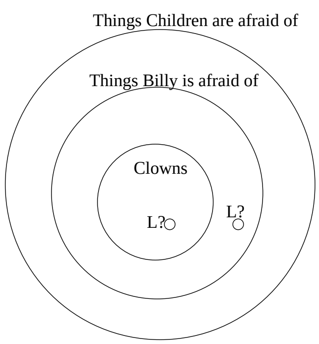
Since we are given a specific truth value assignment, we do not need to build the entire truth table. We can either compute the truth value of the final statement by applying the rules for each operation, or we can build a single row of the truth table (the row corresponding to the specific truth value given).

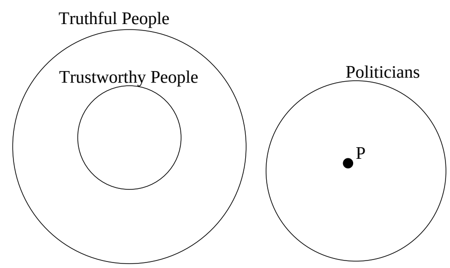
p	q	r	s	$\sim q$	$p \vee \sim q$	$\sim r$	$\sim r \wedge s$	$(p \vee \sim q) \rightarrow (\sim r \wedge s)$
F	T	F	T	F	F	T	T	T

9. (6 points each) Use Euler diagrams to determine whether the following syllogisms are valid or invalid:

(a) All children are scared of clowns.
Billy is a child.
Billy is scared of Lenny
Therefore, Lenny is a clown.

(b) No politicians tell the truth.
Everyone who can be trusted tells the truth.
Therefore, no politicians can be trusted.





Invalid

Valid

10. (6 points each) Define variables and translate each of the following arguments into symbolic form. Then identify the form of each argument and state whether or not the given argument is valid:

(a) If you wish upon a star, then your dreams will come true. Your dreams did not come true. Therefore, you did not wish upon a star.

Using the variables: p : I wish upon a star; q : My dreams come true.
The form of this argument is:

$p \rightarrow q$
$\sim q$
$\therefore \sim p$

The is the Law of Contraposition, which is **Valid**.

(b) If you wish upon a star, then your dreams will come true. Your dreams came true. Therefore, you wished upon a star.

Using the variables: p : I wish upon a star; q : My dreams come true.
The form of this argument is:

$p \rightarrow q$
q
$\therefore p$

The is the Fallacy of the Converse, which is **Invalid**.

11. (12 points) First, translate the following argument into symbolic form. Then, use a truth table to determine whether or not the argument is valid.

If I get a new job then I will go on vacation.
I will buy a new car or I will go on vacation.
I did not buy a new car.

Therefore, I got a new job.

Using the variables p : I get a new job; q : I go on vacation; r : I buy a new car, the symbolic form of this argument is:

$$\begin{array}{l} p \to q \\ r \vee q \\ \underline{\sim r} \\ \therefore p \end{array}$$

Therefore, to analyze the validity of this argument, we will look at the truth table for the statement:

$[(p \to q) \wedge (r \vee q) \wedge (\sim r)] \to p$

p	q	r	$p \to q$	$r \vee q$	$(p \to q) \wedge (r \vee q)$	$\sim r$	$(p \to q) \wedge (r \vee q) \wedge \sim r)$	p	$(p \to q) \wedge (r \vee q) \wedge (\sim r) \to p$
T	T	T	T	T	T	F	F	T	T
T	T	F	T	T	T	T	T	T	T
T	F	T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	F	T	T
F	T	T	T	T	T	F	F	F	T
F	T	F	T	T	T	T	T	F	F
F	F	T	T	T	T	F	F	F	T
F	F	F	T	F	F	T	F	F	T

Notice that there is a False entry in the last column. Therefore, this argument is *invalid*.

12. (10 points) Given the argument:

$p \to q$
$\sim p \to s$
$t \vee \sim q$
$\sim t$
$\therefore\ s$

Fill in the missing statements and reasons in the following two column proof:

1. $t \vee \sim q$	Premise
2. $\sim t$	Premise
3. $\sim q$	1, 2, Disjunctive Syllogism
4. $p \to q$	Premise
5. $\sim q \to \sim p$	4, Contraposition
6. $\sim p \to s$	Premise
7. $\sim q \to s$	5, 6, Law of Syllogism
8. s	3, 7, Law of Detachment