Math 102
Exam 3 - Version 1
03/28/2012

Name: ______________________________

1. (4 points each) Let $U = \{a, b, c, d, e, f, g, h\}$; $A = \{a, c, d, g\}$; $B = \{c, d, f, g\}$; $C = \{a, b, c, e\}$. Give each of the following sets in roster notation:

(a) $C - A$

$C - A = \{b, e\}$

$A \cap B$

$= A \cap B = \{c, d, g\}$

(c) $(C \cup A')$

$A' = \{b, e, f, h\}$

$(C \cup A') = \{a, b, c, e, f, h\}$

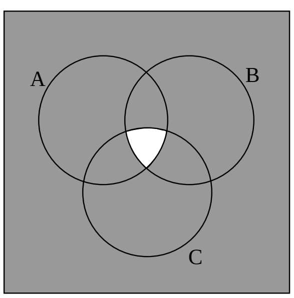
(d) $(B \cap C)'$

$B \cap C = \{c\}$

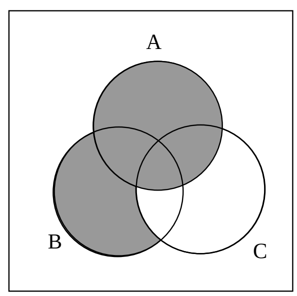
$(B \cap C)' = \{a, b, d, e, f, g, h\}$

2. (5 points each) Illustrate the following by shading the appropriate regions of the given Venn diagrams:

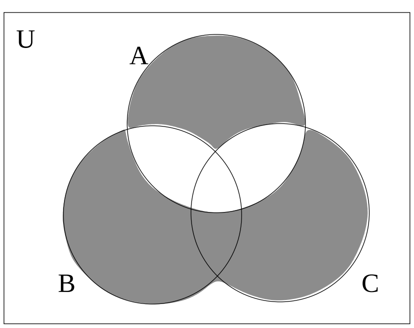
(a) $(A \cap B \cap C)'$



(b) $(B - C) \cup A$

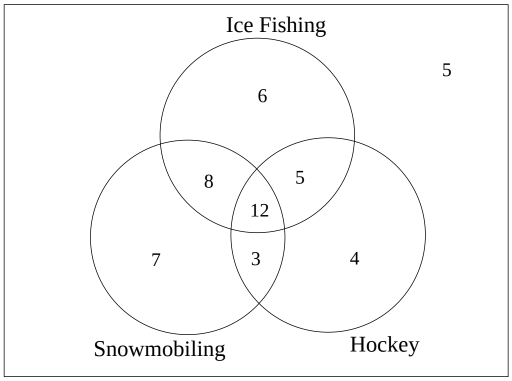


3. (5 points) Use set notation to describe the regions shaded in the Venn diagram given below:



$(A \cup B \cup C) - [(A \cap B) \cup (A \cap C)]$ or $[(B \cup C) - A] \cup [A - (B \cup C)]$

4. (10 points) A survey asked 50 people about winter activities that they enjoy. Specifically, they were asked whether they typically spend time ice fishing, snowmobiling, or playing hockey. Suppose the survey found that 31 of the people surveyed go ice fishing, 30 go snowmobiling, 7 *only* go snowmobiling, 12 do all three, 15 participate in both snowmobiling and hockey, 17 people participate in both hockey and ice fishing, and 7 play hockey but *do not* go ice fishing.



(a) How many do none of the three activities?

5 people do none of the three activities

(b) How many people play hockey?

24 people play hockey.

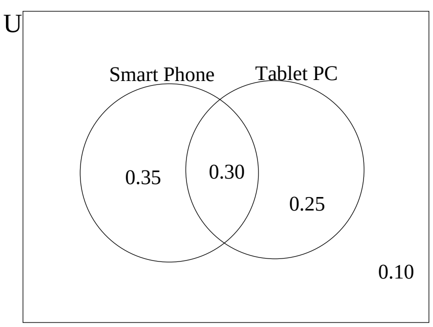
(c) How many only go ice fishing?

6 people only go ice fishing.

(d) How many snowmobile but do not play hockey?

15 people snowmobile but do not play hockey.

5. A survey finds that 65 percent of Americans own a smart phone, 55 percent own a tablet PC, and 30 percent own both a smart phone and a tablet PC.



(a) (4 points) Find the probability that someone owns a tablet PC **or** a smart phone.

$P(SP \cup PC) = 0.90$ or 90%.

(b) (4 points) Find the probability that a person owns *neither* a tablet PC *nor* a smart phone.

$P((SP \cup PC)') = 0.10$ or 10%.

(c) (4 points) *Given* that a person owns a tablet PC, find the probability that this person *also* owns a smart phone.

$P(SP|PC) = \frac{P(SP \cap PC}{P(PC)} = \frac{.30}{0.55} = \frac{30}{55} = \frac{6}{11}$ or approximately 54.54%.

(d) (4 points) Are owning a computer and owning a cell phone independent? Justify your answer.

No. Notice that $P(SP) = 0.65$ while $P(SP|PC) \approx 54.54\%$. Since knowing that a person has a Tablet PC changes the probability that they have a Smart Phone, these events are not independent.

6. A bag contains 5 red balls, 2 green balls, and 3 white balls.

(a) Suppose **one** ball is randomly drawn from the bag (assume each ball is equally likely to be drawn).

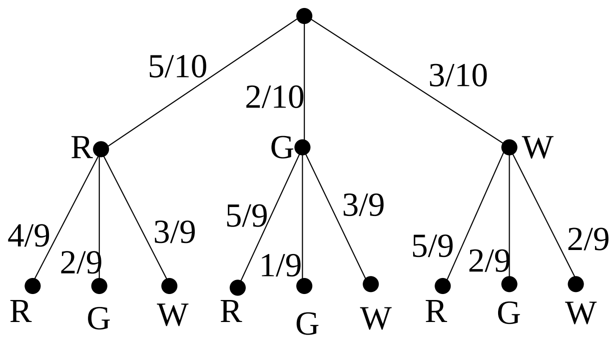
i. (5 points) Find the **probability** of drawing a green ball.

Since there are 10 balls, 2 of which are green, $P(G) = \frac{2}{10} = \frac{1}{5}$ or 20%.

ii. (5 points) Find the **odds against** drawing a white ball.

Notice that there are 3 white balls and 7 non-white balls. Then the odds against drawing a white ball are 7 : 3.

(b) Now suppose that all 10 balls are returned to the bag and then **two** balls are randomly drawn from the bag, one at a time, *without replacement.*



i. (5 points) Find the probability that *both* balls are red.

$P(R, R) = \frac{5}{10} \cdot \frac{4}{9} = \frac{1}{2} \cdot \frac{4}{9} = \frac{2}{9}$ or approximately 22.2%.

ii. (5 points) Find the probability that *neither* ball is green.

There are two main ways to do this problem.
One way is to find the sum of the probabilities for the cases where a non-green ball is chosen at each step:
$P(R, R) + P(R, W) + P(W, R) + P(W, W)$.

A slightly simpler way to compute this is to notice that there are initially 8 non-green balls, so:
$P(G', G') = \frac{8}{10} \cdot \frac{7}{9} = \frac{4}{5} \cdot \frac{7}{9} = \frac{28}{45}$ or approximately 62.2%.

iii. (5 points) Find the probability that one ball is green and the other is red.

Notice that there are two ways of choosing two balls so that one ball is green and the other is red.
$P(R, G) + P(G, R) = \frac{1}{2} \cdot \frac{2}{9} + \frac{1}{5} \cdot \frac{5}{9} = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$ or approximately 22.2%.

7. Suppose that 2 cards are drawn (at the same time) from a standard deck of 52 cards:

   (a) (5 points) Find the number of outcomes in the sample space.

   I accepted two answers to this question.
   If you viewed the two cards as being taken as an unordered subset, then $n(S) = C(52, 2) = 1326$.
   If you viewed the two cards as being taken one at a time as a ordered collection, then $n(S) = P(52, 2) = 2652$.

   (b) (5 points) Find the probability that both cards are spades.

   Recall that 13 of the 52 cards in the deck are spades.
   Using unordered counting, $P(S_1, S_2) = \frac{C(13,2)}{C(52,2)} = \frac{78}{1326} = \frac{1}{17}$ or approximately 5.9%.

   Using ordered counting, $P(S_1, S_2) = \frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$

   (c) (5 points) Find the probability that **exactly one** of the cards is a spade.

   Recall that 13 of the 52 cards in the deck are spades, so the other 39 are not spades.
   Then $P(S, S') = \frac{C(13,1)C(39,1)}{C(52,2)} = \frac{13 \cdot 39}{1326} = \frac{507}{1326} = \frac{13}{34}$ or approximately 38.2%.

8. Suppose I offer to let you play the following game: You pay \$10 for the opportunity to draw a single card from a standard 52 card deck. If you draw a spade, then you win \$42 (your original \$10 plus \$32 more). Otherwise, you lose your \$10.

   (a) (5 points) Find the expected value for playing this game (round your answer to the nearest penny). Is it worth your while to play this game?

   Recall that there are 13 spades in the deck. Therefore, there are 39 non-spades.
   From this, the expected value for this game is:
   $E.V. = \frac{1}{4}(\$32) + \frac{3}{4}(-\$10) = \frac{32-30}{4} = \frac{2}{4} = \frac{1}{2} = 0.50.$
   Therefore, you expect to win approximately 50 cents per play (on average), so this game *is* worth playing.

   (b) (5 points) Suppose that, unbeknownst to you (but knowst to me), I removed the Ace of Spades from the deck before the start of the game (leaving 51 cards in the deck). Find the expected value for playing this game under these new circumstances (round your answer to the nearest penny). Would it be worth your while to play this game now?

   Recall that there were 13 spades in the deck. Since I removed one, there are only 12 left. There are still 39 non-spades.
   From this, the expected value for this game is:
   $E.V. = \frac{12}{51}(\$32) + \frac{39}{51}(-\$10) = \frac{384-390}{51} = \frac{-6}{51} \approx -0.118.$
   Therefore, you expect to lose approximately 12 cents per play (on average), so this game *is not* worth playing.