Math 327
Chapter 6: Equivalent Conditions

Note: We now know that the following statements are all equivalent for an $n \times n$ matrix A:

- A is non-singular.
- $A\vec{x} = \vec{0}$ has only the trivial solution.
- A is row (column) equivalent to I_n.
- For every $\vec{b}$ in R^n, $A\vec{x} = \vec{b}$ has a unique solution.
- A is the product of elementary matrices.
- $det(A) \neq 0$
- rank $A = n$
- The rows (columns) of A form a linearly independent set of n vectors in R_n (R^n).
- The dimension of the solution space of $A\vec{x} = \vec{0}$ is zero.
- The linear transformation $L : R^n \to R^n$ defined by $L(\vec{x}) = A\vec{x}$ for $\vec{x} \in R^n$ is one to one and onto.

Note: We also know that the following statements are all equivalent for a linear transformation $L : V \to W$ between two n dimensional vector spaces:

- L is invertible.
- L is one-to-one.
- L is onto.