Math 327
Exam 2 - Practice Problems

1. For each of the following matrices, determine whether it is in row echelon form, reduced row echelon form, or neither.

(a) $\begin{bmatrix} 1 & -4 & 2 & 0 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -8 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

2. Put each of the following matrices into *row echelon form.*

(a) $\begin{bmatrix} 3 & -2 & 4 & 7 \\ 2 & 1 & 0 & -3 \\ 2 & 8 & -8 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -1 & 2 & 4 & 1 \\ 2 & 1 & 3 & -1 & 2 \\ 1 & 2 & 3 & -2 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

3. Put each of the matrices from the previous problem into *reduced row echelon form.*

4. Use Gaussian Elimination to find all solutions to the following systems of linear equations.

(a) $\begin{cases} x + 2y + 3z = 9 \\ 2x - 2z = -2 \\ 3x + 2y + z = 7 \end{cases}$

(b) $\begin{cases} x + 2y + 3z = 9 \\ 3x + 2y + z = 7 \end{cases}$

5. Use Gauss-Jordan reduction to find all solutions to the following systems of linear equations.

(a) $\begin{cases} 3x - 2y + z = -6 \\ 4x - 3y + 3z = 7 \\ 2x + y - z = -9 \end{cases}$

(b) $\begin{cases} 3x - 2y + z = 4 \\ x + 3y - z = -3 \\ 4x - 10y + 4z = 10 \end{cases}$

6. Use quadratic interpolation to find a quadratic function passing through the points $(0, 2)$, $(1, 0)$, and $(2, 4)$.

7. Construct a quadratic polynomial $p(x) = ax^2 + bx + c$ satisfying the conditions: $p(1) = f(1)$, $p'(1) = f'(1)$, and $p''(1) = f''(1)$ if $f(x) = x \ln x + x^2$.

8. Find the elementary matrices corresponding to carrying out each of the following elementary row operations on a 3×3 matrix:

(a) $r_2 \leftrightarrow r_3$ (b) $-\frac{1}{4}r_2 \to r_2$ (c) $3r_1 + r_2 \to r_2$

9. Find the inverse of each of the elementary matrices you found in the previous problem.

10. Write the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as a product of elementary matrices. [Hint: what elementary operations are needed to put A into reduced row echelon form]

11. For each of the following matrices, either find the inverse or prove that the matrix is singular.

(a) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -5 \\ -1 & \frac{5}{3} \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 0 & 4 \\ 2 & -1 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} -3 & 0 & 4 \\ 2 & -1 & 0 \\ 5 & -1 & 4 \end{bmatrix}$

12. Find all values of a for which the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & a \\ 0 & 2 & -a \end{bmatrix}$ has an inverse. Find the form of A^{-1} (in terms of a) for the cases where it exists.

13. Given the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ -1 & 2 & -5 \end{bmatrix}$:

   (a) Find matrix B equivalent to A having the form described by Theorem 2.12

   (b) Find matrices P and Q such that $B = PAQ$ (for the matrices A and B from above).

14. For each permutation given below, first find the number of inversions. Then, classify the permutation as either even or odd.

   (a) 53214 (b) 45132 (c) 34152

15. Find the determinant of each of the following matrices:

   (a) $\begin{bmatrix} 2 & -4 \\ 5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 & -2 \\ 2 & 1 & 4 \\ 0 & 8 & -5 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} t & 3 \\ 2 & t+1 \end{bmatrix}$

16. For which values of t is the matrix in part (d) above singular? Justify your answer.

17. Evaluate each of the following:

   (a) $\begin{vmatrix} 4 & -2 & 3 & -5 \\ 0 & 1 & 2 & 12 \\ 0 & 0 & -1 & 18 \\ 0 & 0 & 0 & 2 \end{vmatrix}$ (b) $\begin{bmatrix} t-1 & -1 & -2 \\ 0 & t-2 & 2 \\ 0 & 0 & t-3 \end{bmatrix}$ (c) $\begin{vmatrix} 4 & -3 & 6 & 1 \\ 4 & 0 & 4 & 1 \\ 4 & 2 & 0 & 1 \\ 4 & 1 & 2 & 1 \end{vmatrix}$

18. Compute the determinant of the following matrix by putting it into triangular form: $A = \begin{bmatrix} 2 & 3 & -1 & 0 \\ 3 & 2 & -4 & -2 \\ 2 & 0 & 1 & 4 \\ 3 & 8 & -8 & -10 \end{bmatrix}$

19. If $det(AB) = 0$, must $det(A)$ or $det(B) = 0$? Justify your answer.

20. Show that if k is a scalar and A is an $n \times n$ matrix, then $det(kA) = k^n det(A)$.

21. Prove Theorem 3.5

22. Prove Theorem 3.8

23. Let $A = \begin{bmatrix} 3 & 1 & 0 & -1 \\ 2 & 0 & -5 & 1 \\ 2 & 1 & -4 & 0 \\ 0 & -1 & 3 & 2 \end{bmatrix}$. Find:

   (a) M_{14} (b) M_{32} (c) $det(M_{32})$ (d) A_{32} (e) A_{41}

24. Find the determinant of the matrix A from the previous problem by using the expansion of $det(A)$ along the last row.