Math 327
Exam 2 Review Sheet

Section 2.1: Echelon Form of a Matrix

• Know the definition of the terms: row echelon form, reduced row echelon form, column echelon form, reduced column echelon form, leading one, elementary row (column) operation, Type I, II, and III operations, row (column) equivalent matrices, pivot, pivot column.
• Be able to recognize whether or not a given matrix is in row (column) echelon form or reduced row (column) echelon form.
• Be able to both identify and carry out elementary row (column) operations of type I, II, and III. Also know the standard symbolic notation used to represent row and column operations.
• Be able to use elementary row operations to put a given matrix into row (column) echelon form or reduced row (column) echelon form.
• Be able to determine whether or not a given pair or matrices are row (column) equivalent to one another.
• Know the statements of Theorems 2.1 and Theorem 2.2 (i.e. know what they say – you don't need to memorize the numbers)

Section 2.2: Solving Linear Systems

• Know how to use Gaussian elimination and back substitution to find all solutions to a system of linear equations.
• Know how to use Gauss-Jordan reduction to find all solutions to a system of linear equations.
• Be able to recognize when a linear system has no solution or has infinitely many solutions based on its reduced row echelon form.
• Be able to solve applications problems involving quadratic interpolation.

Section 2.3: Elementary Matrices; Finding A^{-1}

• Understand the definition of an elementary matrix of Type I, II, or III. Also be able to find the elementary matrix corresponding to a specific elementary row or column operation.
• Know the statements of Theorems 2.5, 2.6, 2.7, 2.8, 2.9, 2.10 and Corollary 2.2
• Know the five equivalent characterizations of a singular matrix from page 120 in your textbook.
• Be able to use elementary row operations to either find the inverse of a given matrix or to show that no inverse exists.

Section 2.4: Equivalent Matrices

• Understand the definition of equivalent matrices.
• Be able to use row and column operations to put a matrix into reduced form (the form guaranteed by Theorem 2.12)
• Given two equivalent matrices A and B, be able to find non-singular matrices P and Q such that $B = PAQ$.
• Understand the statement and proof of Theorem 2.14

Section 3.1: Definition of the Determinant

• Understand the definition of the following: permutation, inversion, odd and even permutations. Also know that there are $n!$ permutations of the set $S = \{1, 2, \cdots, n\}$, half of which are odd and half of which are even.
• Be able to determine the number of inversions in a given permutation and classify it as even or odd.
• Know the definition of the determinant of a matrix and be able to apply the definition to find the determinant of 2×2 and 3×3 matrices, as well as 4×4 and 5×5 matrices having most entries equal to zero.

Section 3.2: Properties of Determinants

• Know the basic properties of the determinant of a matrix as stated in Theorems 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, and 3.9
• In particular, know impact of Type I, II, and III elementary operations on the determinant of a matrix.
• Be able to find the determinant of a matrix by applying one of the theorems above or by reducing the matrix to triangular form.
• Understand and be able to reproduce the proofs of Theorem 3.3, Theorem 3.5, Theorem 3.8, and Corollary 3.2. Also be able to prove similar results using the Theorems from this section.

Section 3.3: Cofactor Expansion

• Know the definition of a submatrix M_{ij}, the associated minor of a_{ij}, and the cofactor A_{ij}.
• Be able to find a submatrix, minor, and cofactor of a given matrix.
• Be able to compute the determinant of a given matrix A by using the expansion of $det(A)$ along the ith row or the expansion along the jth column.
• Be able to combine cofactor expansions with the results of section 3.2 to compute the determinant of a given matrix.