Math 327
Exam 3 - Practice Problems

1. For each of the following matrices, find the adjoint.

(a) $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 4 & 3 \\ 2 & -1 & 5 \\ 2 & 6 & 16 \end{bmatrix}$

2. For each matrix from problem 1, find $A(adj A)$.

3. For each matrix from problem 1, either find A^{-1} or show that A is singular.

4. (a) Find the adjoint of A if $A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$

(b) Find A^{-1}

5. Prove that if A is singular, then $A(adj A) = 0$

6. Use Cramer's Rule to solve each of the following linear systems.

(a) $\begin{cases} x + y = 2 \\ 2x - y = 10 \end{cases}$ (b) $\begin{cases} x - 4y = 6 \\ 3x + y = 5 \end{cases}$ (c) $\begin{cases} 3x - 2y + z = -6 \\ 4x - 3y + 3z = 7 \\ 2x + y - z = -9 \end{cases}$

7. For each given pair of points P and Q, find the vector $\overrightarrow{PQ}$ and then sketch this vector.

(a) $P(-1, 2)$, $Q(3, -5)$ (b) $P(-1, 0, 3)$, $Q(1, 2, 4)$ (c) $P(3, -4, 1)$, $Q(-1, 4, 0)$

8. Determine the tail of the vector $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$ if:

(a) The head is $(1, 2, 3)$ (b) The head is $(3, -2, 0)$ (c) Find the head if the tail is $(1, 2, 3)$

9. Let $\vec{u} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}$. Find:

(a) $\vec{u} - \vec{v}$ (b) $3\vec{u} + 4\vec{v}$ (c) $\vec{w}$ if $\vec{u} + \vec{v} + \vec{w} = \vec{0}$.

10. If possible, find scalars c_1, c_2 and c_3 not all zero such that $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} -4 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

11. If possible, find scalars c_1, c_2 and c_3 not all zero such that $c_1 \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} + c_3 \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

12. Let V be the set of all functions of the form $f(x) = re^{kx}$, where r, k are real numbers. Let $\oplus$ be defined as $r_1e^{k_1x} \oplus r_2e^{k_2x} = r_1r_2e^{(k_1+k_2)x}$ Let $\odot$ be defined as $c \odot re^{kx} = cre^{kx}$ for any real number c. Determine whether or not V is a vector space. If it is, prove that it satisfies each part of the definition of a vector space. If not, show which properties are not satisfied.

13. Let V be the set of all ordered triples (x, y, z) where x, y and z are real numbers. Let $\oplus$ be defined as $(x_1, y_1, z_1) \oplus (x_2, y_2, z_2) = (x_1 + z_2, y_1 + y_2, z_1 + x_2)$ Let $\odot$ be defined as $c \odot (x, y, z) = (cx, cy, cz)$ for any real number c. Determine whether or not V is a vector space. If it is, prove that it satisfies each part of the definition of a vector space. If not, show which properties are not satisfied.

14. Let V be the set of real numbers. Let $\oplus$ be defined as $r \oplus s = rs$. Let $\odot$ be defined as $c \odot r = c + r$ for any real number c. Determine whether or not V is a vector space. If it is, prove that it satisfies each part of the definition of a vector space. If not, show which properties are not satisfied.

15. Prove that a vector space has only one zero vector (that is, the zero of a vector space is unique).

16. Prove that in a vector space, $-1 \odot \vec{u} = -\vec{u}$ for any vector $\vec{u} \in V$.

17. Prove that in any vector space, for a given vector $\vec{u}$, $-(-\vec{u}) = \vec{u}$.

18. Let $V = R^3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where x , y and z are real numbers, and $\oplus$ and $\odot$ are the usual operations. Determine which of the following are subspaces of V:

(a) $W_1 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : 2x - y = z \right\}$

(b) $W_2 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y - z = 0 \right\}$

(c) $W_3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y = 1 \right\}$

19. Let $V = M_{33}$. Determine which of the following are subspaces of V.

(a) W_1 is the set of all 3×3 scalar matrices.
(b) W_2 is the set of all non-singular 3×3 matrices.
(c) W_3 is the set of all symmetric 3×3 matrices.

20. Let V be $C(-\infty, \infty)$ with the usual operations. Determine which of the following are subspaces of V.

(a) W_1 is the set of all continuous functions such that $f(0) = 0$.
(b) W_2 is the set of all continuous function such that $f(0) = 1$.
(c) W_3 is the set of all differentiable functions.
(d) W_4 is the set of all constant functions.

21. Let $v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Which of the following vectors are linear combinations of v_1 and v_2?

(a) $\begin{bmatrix} 6 \\ -12 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

22. Let $v_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix}$. Which of the following vectors are linear combinations of v_1, v_2 and v_3?

(a) $\begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

23. If possible, find a non-zero vector in the null space of each of the following vectors:

(a) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 0 & 4 \\ 2 & -1 & 0 \\ 5 & 0 & -2 \end{bmatrix}$

24. Describe the set of **all** vectors in the null space of the matrix $A = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 0 & -1 \\ 1 & 1 & 3 \end{bmatrix}$