Math 327

Exam 3 Review Sheet

Section 3.4: The Inverse of a Matrix

• Know the statement and proof of Theorem 3.11

• Know the definition of the Adjoint of an $n \times n$ matrix A and be able to use cofactors to compute the adjoint of a given matrix.

• Know the statement of Theorem 3.12 [$A(\text{adj } A) = (\text{adj } A)A = \det(A)I_n$].

• Be able to use the adjoint of a non-singular matrix A to find A^{-1}.

• Know the statement of Corollary 3.4.

Sections 3.5 and 3.6: Other Applications of Determinants, Determinants from a Computational Point of View

• Know the statement of Theorem 3.13 (Cramer's Rule).

• Be able to use Cramer's Rule to find the solution of a linear system of n equations in n unknowns whose coefficient matrix in non-singular.

• Know that Cramer's Rule cannot be applied to a linear system whose coefficient matrix is not square or one whose coefficient matrix is singular.

• Understand that using permutations directly and using the cofactor expansion are computationally inefficient methods of computing the determinant of a matrix, while using Gaussian elimination is much more efficient. From this, using Gaussian Elimination or the Gauss-Jordan method is more computationally efficient than using Cramer's rule to solve a system of equations.

Section 4.1: Vectors in the Place and in 3-Space

• Understand the rectangular coordinate systems for both 2 and 3 dimensional space including the origin O, the definition of the x, y, and z-axes, plotting points in both 2 and 3-dimensional space, and the difference between right-handed and left-handed coordinate systems in 3-space.

• Understand the definition of scalars and vectors. Be able to graph a vector as a directed line segment and be able to identify the head and tail of a vector.

• Understand the definition of the magnitude of a vector, the zero vector, the components of a vector, and equality of two vectors.

• Be able to find sums, differences, and scalar multiples of vectors. Also be able to determine whether or not a given vector is a linear combination of other given vectors.

Section 4.2: Vector Spaces

• Memorize the definition of a real vector space as given in definition 4.4. In particular, know what it means for a set to be *closed* under a given operation.

• Memorize the key examples of vector spaces discussed in class, including how vectors are defined and how the operations $\oplus$ and $\odot$ are defined for each example. Also know the standard notation used to represent these example vector spaces (R^n, R_n, M_{mn}, P, P_n, $C(-\infty, \infty)$).

• Be able to prove that a given example is a vector space by verifying that it satisfies the properties from definition 4.4.

• Be able to prove that a given example is not a vector space by providing specific counterexamples that show that one or more of the properties from definition 4.4 are not satisfied.

• Know the statement of Theorem 4.2 and be able to prove one or more of these properties.

• Know that in a vector space, the vector $\vec{0}$ is unique and that for a given vector $\vec{u}$, its negative $-\vec{u}$ is unique. Also know how to prove these facts.

Section 4.3: Subspaces

• Memorize the definition of a subspace of a vector space.

• Know the statement of Theorem 4.3 and be able to apply it to show that a subset of a vector space is a subspace.

• Be able to show that a given subset of a vector space is not a subspace by providing a specific counterexample to one of the closure properties.

• Know that every vector space has at least two subspaces: the space itself and the zero subspace.

• Know the definition of a linear combination of vectors in a given vector space.

• Be able to determine whether or not a given vector is a linear combination of a set of vectors in a given vector space.

• Know the definition of the null space of a matrix (the solution space of the associated homogeneous system of equations).

• Be able to show that a given vector or a collection of vectors is in the null space of a matrix. Also be able to describe all vectors that are in the null space of a given matrix.