Math 327

Exam 4 Review Sheet

Section 4.4: Span

• know the definition of a linear combination of vectors in a vector space.
• Know the definition of the span of a set of vectors and the definition of a spanning set for a vector space.
• Be able to determine whether or not a given vector is in the span of a set of vectors.
• Be able to determine whether of not a set of vectors spans a given vector space and be able to find a spanning set for a given space.
• Know the statement and proof of Theorem 4.4.

Sections 4.5: Linear Independence

• Know the definition of a linearly dependent set and the definition of a linearly independent set.
• Be able to determine whether or not a given set of vectors is linearly independent.
• Given a linearly dependent set, be able to write one of the vectors as a linear combination of the other vectors in the set.
• Know the statements of Theorems 4.5, 4.6, and 4.7

Section 4.6: Basis and Dimension

• Know the definition of a basis for a vector space. Also be able to determine whether or not a set of vectors is a basis for a vector space.
• Know the definition of the dimension of a vector space. Also know the definitions of a minimal spanning set and a maximal linearly independent set.
• Know the standard bases for R^n, R_n, P_n, P, and M_{mn} as well as the dimension of these vector spaces.
• Know the statements of Theorems 4.8, 4.9, 4.10, 4.11, and 4.13 and the statements of Corollaries 4.1, 4.4, and 4.5.
• Know the proof of Theorem 4.8. Also be able to algorithm from the proof to 4.9 to find a basis starting from a spanning set for a vector space.
• Be able to find a basis for a vector space that contains a given linearly independent set of vectors.
• Be able to find a basis for a subspace of a vector space.

Section 4.7: Homogeneous Systems

• Be able to find a basis for the solution space of a homogeneous system of linear equations.
• Be able to find a basis for the null space of a given matrix.
• Be able to find the dimension of the null space of a vector space. (Note: this dimension is called the **nullity** of the matrix).
• Given a system of the form $(\lambda I_n - A)\vec{x} = \vec{0}$, be able to find all values λ such that the system has a non-trivial solution.

Section 4.8: Coordinates and Isomorphisms

• Know the definition of an ordered basis for a vector space and the definition of $[\vec{v}]_S$, the coordinate vector of a vector $\vec{v}$ with respect to an ordered basis S.
• Be able to find the coordinates of a vector $\vec{v}$ with respect to an ordered bases S.
• Know the definition of a one-to-one map, an onto map, and the definition of an isomorphism between two vector spaces.
• Be able to verify whether or not a given map is an isomorphism.
• Know the statements of Theorems 4.14,4.15, 4.16, and Corollary 4.6.
• Given two bases S and T for a vector space V, be able to find $P_{S \leftarrow T}$, the transition matrix from T to S.
• Be able to use a transition matrix $P_{S \leftarrow T}$ to compute the S coordinates of a vector from the T coordinates of the vector.