Math 327
Exam 5 - Practice Problems

1. Find a basis for the row space of each of the following matrices. Your basis should consist of rows of the original matrix.

(a) $\begin{bmatrix} 1 & -2 & 5 & 4 \\ 2 & 1 & -3 & 7 \\ 1 & -7 & 18 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 & -4 & 1 \\ 0 & 4 & -5 & 2 \\ 2 & -1 & 1 & 5 \\ 2 & 3 & -4 & 7 \\ 3 & 2 & -3 & 6 \end{bmatrix}$

2. Find a basis for the column space of each of the matrices from problem 1. Your basis should consist of columns of the original matrix.

3. Find the rank of each of the matrices from problem 1.

4. Find a basis for the null space of each of the matrices from problem 1.

5. Use the determinant to determine whether or not each of the following matrices has rank 3:

(a) $\begin{bmatrix} -2 & 3 & 1 \\ 0 & -1 & 2 \\ 2 & 1 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 3 & 1 \\ 0 & 4 & 7 \\ 2 & 1 & 6 \end{bmatrix}$

6. Use the rank of the coefficient matrix to determine whether or not each of the following homogeneous systems have a non-trivial solution.

(a) $A\vec{x} = \vec{0}$ if $A = \begin{bmatrix} 5 & -1 & 2 & 4 \\ 2 & -3 & 0 & 1 \\ 1 & -5 & 3 & 7 \\ 0 & 2 & -3 & 1 \end{bmatrix}$

(b) $A\vec{x} = \vec{0}$ if $A = \begin{bmatrix} 5 & -1 & 2 & 4 \\ 2 & -3 & 0 & 1 \\ 1 & -5 & 3 & 7 \\ 0 & -7 & 6 & 13 \end{bmatrix}$

7. Suppose that A is a 4×7 matrix.

(a) What is the maximum rank of A?

(b) Could the columns of A be linearly independent? Justify your answer.

(c) Could the rows of A be linearly independent? Justify your answer.

(d) If the rank of A is 3, find the nullity of A.

(e) If the rank of A is 3, find the nullity of A^T.

8. Let A be an $n \times n$ matrix. Show that rank $A = n$ if and only if the columns of A are linearly independent.

9. Consider the following functions from $R^3 \to R^2$:

$$L_1\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ u_3 \end{bmatrix} \qquad L_2\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ 0 \end{bmatrix}$$

$$L_3\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ 1 \end{bmatrix} \qquad L_4\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ u_3^2 \end{bmatrix}$$

(a) Determine whether or not each of these functions is a linear transformation. Justify your answer.

(b) For each L_i this is a linear transformation, find a matrix representing the linear transformation.

(c) For each L_i this is a linear transformation, find the kernel and range of the linear transformation.

10. Suppose that L is a linear transformation with:

$$L\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \; L\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \text{ and } L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

(a) Find $L\left(\begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}\right)$.

(b) Find the standard matrix representing this linear transformation.

(c) Determine whether or not L is one-to-one.

11. Let $L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 + u_2 + u_3 \\ 0 \end{bmatrix}$

(a) Prove that L is a linear transformation.

(b) Show that L is not one-to-one and find a basis for $ker\, L$.

(c) Determine whether or not L is onto.

12. Let $L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 + u_2 \\ u_3 - u_2 \end{bmatrix}$

(a) Prove that L is a linear transformation.

(b) Show that L is not one-to-one and find a basis for $ker\, L$.

(c) Determine whether or not L is onto.

13. Prove Theorem 6.1(a)

14. Prove Theorem 6.4(b)

15. Given a linear transformation $L : V \to W$, suppose that $dim\, V = n$ and $dim\, W = m$ with $m > n$.

(a) Could L be one-to-one? Justify your answer.

(b) Could L be onto? Justify your answer.

(c) Could L be invertible? Justify your answer.

16. Consider the matrix: $\begin{bmatrix} 4 & -1 \\ 3 & 0 \end{bmatrix}$

(a) Find the characteristic polynomial for each of this matrix.

(b) Find the eigenvalues for this matrix.

(c) For each eigenvalue, find an associated eigenvector.

17. Consider the matrix: $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 3 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

(a) Find the characteristic polynomial for each of this matrix.

(b) Find the eigenvalues for this matrix.

(c) For each eigenvalue, find an associated eigenvector.