Math 327
Linear Systems of Equations
Definitions and Notation Handout

Definitions:

- A **linear equation in n variables** is an equation of the form: $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$. Here, $x_1, x_2, \cdots, x_n$ are unknowns while $a_1, a_2, \cdots, a_n, b$ are constants.
- A **solution** to a linear equation is an assignment of values to the unknowns: $x_1 = s_1, x_2 = s_2, \cdots, x_n = s_n$ so that the equation is satisfied.
- A **system of m linear equations in n unknowns**, $x_1, x_2, \cdots, x_n$ or, more concisely, a **linear system**, is a set of m linear equations in n unknowns.
- A **solution** to a linear system is an assignment of values to the unknowns: $x_1 = s_1, x_2 = s_2, \cdots, x_n = s_n$ so that all m of the equations are satisfied simultaneously.
- A linear system that has no solution is said to be **inconsistent**.
- A linear system that has a solution is said to be **consistent**.
- In the special case where the constants $b_1 = b_2 = \cdots = b_n = 0$, we say that the linear system is a **homogeneous system**.
  - Notice that every homogeneous system of equations is satisfied by the **trivial solution**: $x_1 = x_2 = \cdots = x_n = 0$.
  - A solution to a homogeneous system of equations in which *not all* x_i are zero is called a **non-trivial solution**.
- Two linear systems are called **equivalent** if they have exactly the same solutions.

Notation: When writing a linear system, we will generally use the following "double subscript" notation:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$