Math 327
Matrices
Definitions and Notation Handout

Definitions:

- An $m \times n$ **matrix** (often denoted by a capital letter A) is a rectangular array of $m \cdot n$ real (or complex) numbers arranged in m horizontal **rows** and n vertical **columns**.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & a_{ij} & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

  The i^{th} row of A is: $\begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}$.

  The j^{th} column of A is: $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$.

  We call A an m by n matrix (written $m \times n$), and we use a_{ij} to refer to the entry of the matrix located within both the i^{th} row and the j^{th} column.

- In the special case where $m = n$, we call A a **square matrix of order** n. In this case, the entries $a_{11}, a_{22}, \cdots, a_{nn}$ form the **main diagonal** of the matrix A.
- Note that an $n \times 1$ matrix is often called an n-vector.

Matrix Operations

1. Equality

   Two matrices A and B are **equal** if A and B are $m \times n$ matrices for which $a_{ij} = b_{ij}$ for $1 \leq i \leq m$ and $1 \leq j \leq n$. That is, A and B are the same size and each pair of their entries agree.

   Example:

2. Addition

   If A and B are both $m \times n$ matrices, we define the sum of these matrices $A + B$ as follows: If $A = [a_{ij}]$ and $B = [b_{ij}]$, then $A + B = [a_{ij} + b_{ij}]$ (that is, each corresponding entry is added together).

   Example:

   Note: If O is the zero matrix of size $m \times n$ (an m by n matrix all of whose entries are zero), then $A + O = A$.

3. Scalar Multiplication

   Given a constant c, we define cA (a scalar multiple of A) as $cA = [c\dot{a}_{ij}]$ (that is, each entry of A is multiplied by c).

   Example:

4. The Transpose of a Matrix

   Given a matrix $A = [a_{ij}]$, the transpose of this matrix, denoted by A^T is defined as $A^T = [a_{ij}]$ (the rows of A become the columns of A^T). Note that is A is an $m \times n$ matrix, then A^T is an $n \times m$ matrix.

   Example:

5. Matrix Multiplication (next time...)

Summation Notation:

Recall that **summation notation** is a compact way of writing repeated addition. For example, $\sum_{i=1}^{5} a_i = a_1 + a_2 + a_3 + a_4 + a_5$.

Here are three properties of summations:

1. $\displaystyle\sum_{i=1}^{n} (r_i + s_i)\, a_i = \sum_{i=1}^{n} r_i a_i + \sum_{i=1}^{n} s_i a_i.$
2. $\displaystyle\sum_{i=1}^{n} c a_i = c \sum_{i=1}^{n} a_i$
3. $\displaystyle\sum_{j=1}^{n} \left(\sum_{i=1}^{m} a_i j \right) = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_i j \right)$