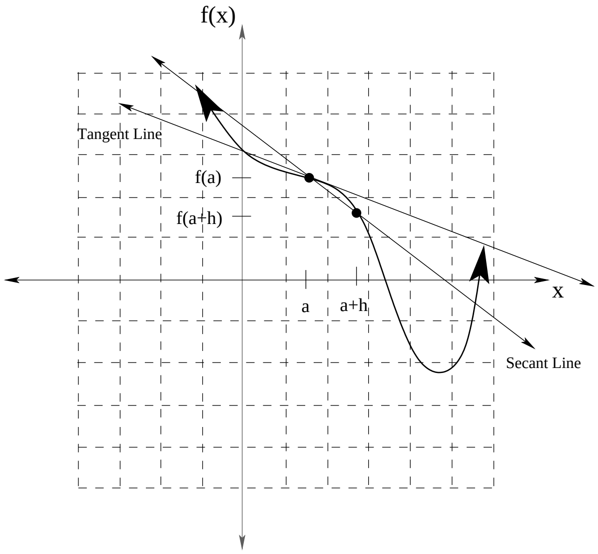
Math 229

Secant Lines and Tangent Lines Handout



• A *secant line* meets the graph of a function at two points. The slope of the secant line through the points $P(a, f(a))$ and $Q(a+h, f(a+h))$ is equal to the *average rate of change* of the function f over the interval $[a, a+h]$.
For example, the slope of a secant line to a position (displacement) graph is the *average speed* of the object being described over the time period between the two points on the graph intersecting the line.

• A *tangent line* meets the graph of a function at a point $P(a, f(a))$, and has slope equal to the *instantaneous rate of change* of the function f at the point P.
For example, the slope of a tangent line to a position (displacement) graph is the *instantaneous velocity* of the object being described at that particular point in time.

• The slope of the tangent line to a function $f(x)$ at the point $P(a, f(a))$ (that is, when $x = a$) is given by:
$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$, when this limit exists.

Example: If $f(x) = -2x^2 + 8x$, and $x = a$, then:

$$m_a = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{-2(a+h)^2 + 8(a+h) - (-2a^2+8a)}{h} = \lim_{h\to 0} \frac{-2(a^2+2ah+h^2)+8a+8h+2a^2-8a}{h} =$$
$$\lim_{h\to 0} \frac{-2a^2-4ah-2h^2+8a+8h+2a^2-8a}{h} = \lim_{h\to 0} \frac{-4ah-2h^2+8h}{h} = \lim_{h\to 0} -4a-2h+8 = -4a+8$$

In particular, if $a = 1$, then $m_a = -4 + 8 = 4$

• To find an equation for a tangent line to a function f at a point $P(a, f(a))$, we use the point P together with the slope m_a.
Example: given the function $f(x) = -2x^2 + 8x$, and $x = 1$, $(1, f(1)) = (1, 6)$, and $m = 4$, so the equation of the tangent line at this point is given by: $y - 6 = 4(x-1)$, or $y = 4x + 2$.