Math 261
Exam 1 - Practice Problems

1. Given the points $A : (-2, 4)$ and $B : (7, -1)$:

   (a) Find an equation for the line containing A and B

   First, we find the slope of the line: $m = \frac{\Delta y}{\Delta x} = \frac{4-(-1)}{-2-7} = \frac{5}{-9} = -\frac{5}{9}$.

   Next, we use the point-slope equation to find the equation for the line: $y + 1 = -\frac{5}{9}(x - 7) = -\frac{5}{9}x + \frac{35}{9}$, so $y = -\frac{5}{9}x + \frac{35}{9} - 1$, or $y = -\frac{5}{9}x + \frac{26}{9}$.

   (b) Find the equation for the line through the origin and perpendicular to $\overleftrightarrow{AB}$

   The slope of the line is the negative reciprocal of our previous line. That is, $m = \frac{9}{5}$.

   Since the line passes through the origin, $(0, 0)$, $b = 0$

   Therefore, the perpendicular line has equation $y = \frac{9}{5}x$

2. Given the equation $4x + 3y = -2$

   (a) Find the slope of the line represented by this equation.

   Solving for y, we get $3y = -4x - 2$, or $y = -\frac{4}{3}x - \frac{2}{3}$, so the slope is $m = -\frac{4}{3}$.

   (b) Find the x and y intercepts for this line.

   From the slope-intercept form found in the work above, the y=intercept is $-\frac{2}{3}$.

   To find the x-intercept, notice that when $y = 0$ the original equation becomes $4x = -2$, so $x = -\frac{1}{2}$ is the x-intercept.

   (c) Give an equation for a line through $(-2, 4)$ that is parallel to this line.

   Since we want a line that is parallel to the original line, the slope is the same as the slope of the original line. That is, $m = -\frac{4}{3}$.

   Using the point slope formula the line we are looking for has equation:

   $y - 4 = -\frac{4}{3}(x + 2) = -\frac{4}{3}x - \frac{8}{3}$, so $y = -\frac{4}{3}x - \frac{8}{3} + 4 = -\frac{4}{3}x - \frac{8}{3} + \frac{12}{3}$

   or, in slope/intercept form: $y = -\frac{4}{3}x + \frac{4}{3}$

3. Suppose you own a company that manufactures widgets. Your supplier sells you the widgets wholesale at \$8 apiece. It costs you \$750 a month to rent your store, and you spend an additional \$2150 each month on utilities, supplies, and employee salaries. You sell the widgets at a retail price of \$15 apiece.

   (a) Find an equation $C(x)$ that gives your monthly costs, where x is the number of widgets you purchase for sale that month.

   The fixed monthly costs are \$2150+ \$750 = \$2900. Therefore, since the widgets cost \$8 each wholesale, the cost function is: $C(x) = 8x + 2900$.

   (b) Find equations for your monthly revenue, $R(x)$, and your monthly profit, $P(x)$, assuming that you sell all of the new widgets that you purchase.

   Since we charge \$15 retail for each widget, $R(x) = 15x$.

   Our profit is given by $P(x) = R(x) - C(x) = 15x - (8x + 2900) = 7x - 2900$.

   (c) How many widgets do you need to sell each month in order to break even?

   We break even when profit is zero, that is, when $7x - 2900 = 0$, or when $7x = 2900$, so $x \approx 414.2$.

   Therefore, we must sell 415 widgets to break even (well, since we rounded up, we would make a slight profit).

(d) Suppose that instead of purchasing your widgets wholesale, you could instead build a factory and make your own widgets for only \$4 apiece. If the factory would cost \$750,000 to build, but all of your other costs would remain the same, how many widgets would you need to sell during the lifetime of the factory in order to make the construction worthwhile?

The monthly cost function for our company with the new factory would be:

$C_2(x) = 4x + 750,000 + 2900 = 4x + 752,900$. Widgets now cost us only \$4 apiece, but we have the same fixed costs, plus the cost of building the factory.

It would make sense to build the factory if the factory costs us less than going with the wholesale supplier. This happens at the intersection our previous and new cost functions.

That is, when $4x + 752,900 = 8x + 2900$, or when $4x = 750,000$. That is, when $x = 187,500$.

Therefore, if we expect to sell over 187,500 widgets over the lifetime of the factory, it would be worth it to build the factory.

4. Simplify the following:

(a) $\left(\frac{3}{4}\right)^{-2}$

$$= \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

(b) $\left(\frac{y^{12}}{25z^4}\right)^{-\frac{3}{2}}$

$$= \left(\frac{25z^4}{y^{12}}\right)^{\frac{3}{2}} = \left(\left(\frac{25z^4}{y^{12}}\right)^{\frac{1}{2}}\right)^3$$

$$= \left(\frac{5z^2}{y^6}\right)^3 = \frac{125z^6}{y^{18}}$$

(c) $\sqrt[5]{32x^{11}y^{14}z^8}$

$$= 2x^2y^2z\sqrt[5]{xy^4z^3}$$

(d) $\left(\frac{(5xyz)^2z^{-2}}{2x^{-2}y^2z^{-4}}\right)^{-1}$

$$= \left(\frac{25x^2y^2z^2x^2z^4}{2y^2z^2}\right)^{-1} = \left(\frac{25x^4z^4}{2}\right)^{-1} = \frac{2}{25x^4z^4}$$

(e) $\frac{3x^2-10x+3}{x^2-1} \cdot \frac{x^2+x-2}{x^2-9}$

$$= \frac{(3x-1)(x-3)}{(x+1)(x-1)} \cdot \frac{(x+2)(x-1)}{(x+3)(x-3)}$$

$$= \frac{(3x-1)(x+2)}{(x+1)(x+3)}$$

(f) $\frac{2x^2+4}{2x^2+7x-4} - \frac{x-1}{x+4}$

$$= \frac{2x^2+4}{(2x-1)(x+4)} - \frac{x-1}{x+4} \cdot \frac{(2x-1)}{(2x-1)} = \frac{(2x^2+4)-(x-1)(2x-1)}{(2x-1)(x+4)}$$

$$= \frac{(2x^2+4)-(2x^2-3x+1)}{(2x-1)(x+4)} = \frac{3x+3}{(2x-1)(x+4)} = \frac{3(x+1)}{(2x-1)(x+4)}$$

(g) $\dfrac{\frac{1}{x}+\frac{3}{x-2}}{\frac{4}{x-1}-\frac{2}{x-2}}$

$$= \frac{\frac{1}{x}+\frac{3}{x-2}}{\frac{4}{x-1}-\frac{2}{x-2}} \cdot \frac{x(x-1)(x-2)}{x(x-1)(x-2)} = \frac{\frac{x(x-1)(x-2)}{x}+\frac{3x(x-1)(x-2)}{x-2}}{\frac{4x(x-1)(x-2)}{x-1}-\frac{2x(x-1)(x-2)}{x-2}}$$

$$= \frac{(x-1)(x-2)+3x(x-1)}{4x(x-2)-2x(x-1)} = \frac{x^2-3x+2+3x^2-3x}{4x^2-8x-2x^2+2x} = \frac{4x^2-6x+2}{2x^2-6x}$$

$$= \frac{2(2x^2-3x+1)}{2x(x-3)} = \frac{2(2x-1)(x-1)}{2x(x-3)}$$

(h) $\dfrac{\frac{3}{2x+2h+1}-\frac{3}{2x+1}}{h}$

$$= \frac{\frac{3(2x+1)}{(2x+2h+1)(2x+1)}-\frac{3(2x+2h+1)}{(2x+2h+1)(2x+1)}}{h} = \frac{6x+3-(6x+6h+3)}{(2x+2h+1)(2x+1)} \cdot \frac{1}{h}$$

$$= \frac{-6h}{(2x+2h+1)(2x+1)} \cdot \frac{1}{h} = \frac{-6}{(2x+2h+1)(2x+1)}$$

5. Rationalize the denominator in the following expressions:

(a) $\dfrac{3x}{\sqrt[3]{x}}$

$$= \frac{3x}{\sqrt[3]{x}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{3x\sqrt[3]{x^2}}{x} = 3\sqrt[3]{x^2}$$

(b) $\dfrac{2x+3}{\sqrt{2x}-1}$

$$= \frac{2x+3}{\sqrt{2x}-1} \cdot \frac{\sqrt{2x}+1}{\sqrt{2x}+1} = \frac{(2x+3)(\sqrt{2x}+1)}{2x-1}$$

6. Factor each of the following expressions completely:

(a) $2x^2+x-6$

$= (2x-3)(x+2)$

(b) $50x^2+45x-18$

$ac = -900$, which factors as $60 \cdot (-15)$

so $50x^2+60x-15x-18 = 10x(5x+6)-3(5x+6) = (10x-3)(5x+6)$

(c) $16x^2-25y^2$

$(4x+5y)(4x-5y)$

(d) $6x^3y-27x^2y-15xy$

$3xy(2x^2-9x-5) = 3xy(2x+1)(x-5)$

(e) $3x^3+x^2-3x-1$

$x^2(3x+1)-(3x+1) = (x^2-1)(3x+1) = (x+1)(x-1)(3x+1)$

7. Given the function $f(x) = \dfrac{2}{x-4}$

   (a) What is the domain of f? Give your answer in interval notation.

   We need the denominator to be non-zero. Therefore, $x \neq 4$ is the domain. In interval notation, this is:

   $(-\infty, 4) \cup (4, \infty)$.

   (b) Find $f(12)$

   $f(12) = \frac{2}{12-4} = \frac{2}{8} = \frac{1}{4}$.

   (c) Find $f(2a+4)$

   $f(2a+4) = \frac{2}{(2a+4)-4} = \frac{2}{2a} = \frac{1}{a}$.

   (d) Find $\dfrac{f(a+h)-f(a)}{h}$. Simplify your answer.

$$\frac{\frac{2}{a+h-4} - \frac{2}{a-4}}{h} = \frac{\frac{2(a-4)-2(a+h-4)}{(a+h-4)(a-4)}}{h} = \frac{2a-8-2a-2h+8}{(a+h-4)(a-4)}\frac{1}{h} = \frac{-2h}{(a+h-4)(a-4)}\frac{1}{h}$$

$$= \frac{-2}{(a+h-4)(a-4)}$$

8. Given that $f(x) = \dfrac{1}{3x-2}$ and $g(x) = \sqrt{x^2-4}$

   (a) Find $\dfrac{g}{f}(x)$

$$\frac{g}{f}(x) = \frac{\sqrt{x^2-4}}{\frac{1}{3x-2}} = (3x-2)\sqrt{x^2-4}.$$

   (b) Find $g \circ f(x)$

$$g(f(x)) = \sqrt{\left(\frac{1}{3x-2}\right)^2 - 4} = \sqrt{\frac{1}{9x^2-12x+4} - 4}$$

   (c) Find $f \circ g(2)$

   $g(2) = \sqrt{2^2-4} = 0$, so $f(g(2)) = f(0) = \frac{1}{3(0)-2} = -\frac{1}{2}$

9. Evaluate the following limits. Be sure to show enough work to justify your answers.

   (a) $\displaystyle\lim_{x\to 0} \frac{x^2-2x}{2x^2-x-6}$

$$\lim_{x\to 0} \frac{x^2-2x}{2x^2-x-6} = \frac{0^2-2(0)}{2(0)^2-0-6} = \frac{0}{-6} = 0$$

   (b) $\displaystyle\lim_{x\to 2} \frac{x^2-2x}{2x^2-x-6}$

   First notice that this limit cannot be evaluated directly, since it leads to the indeterminate form $\frac{0}{0}$. Therefore, we try simplifying using algebra:

$$\lim_{x\to 2} \frac{x^2-2x}{2x^2-x-6} = \lim_{x\to 2} \frac{x(x-2)}{(2x+3)(x-2)} = \lim_{x\to 2} \frac{x}{2x+3} = \frac{2}{2(2)+3} = \frac{2}{7}$$

(c) $\lim_{x \to 2} \dfrac{x}{x-2}$

First notice that this limit cannot be evaluated directly, since it leads to the form $\frac{0}{-2}$, which leads us to suspect that this limit might not exist. To verify this, we investigate by evaluating the expression inside the limit at points close to 2.

x	1.9	2.1	1.99	2.01	1.999	2.001
$f(x)$	-19	21	-199	201	-1999	2001

Since the values are diverging as we get closer and closer to 2, we can conclude that the limit does not exist.

(d) $\lim_{x \to \infty} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

For this limit, since we are looking at the limit as it approaches positive infinity, we need only consider the highest order terms:

$$\lim_{x \to \infty} \frac{x^2 - 2x}{2x^2 - x - 6} = \lim_{x \to \infty} \frac{x^2}{2x^2} = \frac{1}{2}$$

(e) $\lim_{x \to 0} \dfrac{2x^2 - x - 1}{x^2 - 1}$

$$\lim_{x \to 0} \frac{2x^2 - x - 1}{x^2 - 1} = \frac{2(0)^2 - (0) - 1}{(0)^2 - 1} = \frac{-1}{-1} = 1$$

(f) $\lim_{x \to 1} \dfrac{2x^2 - x - 1}{x^2 - 1}$

Here, just plugging in 1 gives us the indeterminate form $\frac{0}{0}$. To see whether or not the limit exists, we need to do further investigation.

Factoring the expression inside the limit gives us:

$\lim_{x \to 1} \dfrac{(2x+1)(x-1)}{(x+1)(x-1)}$, which, canceling the like terms in the fraction, gives

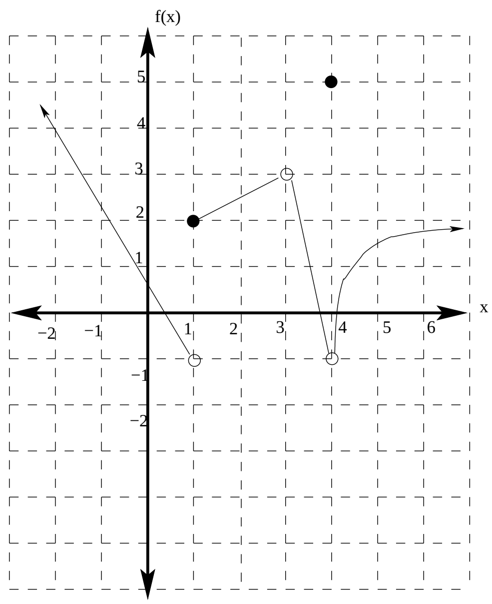
$$\lim_{x \to 1} \frac{2x+1}{x+1} = \frac{2(1)+1}{1+1} = \frac{3}{2}.$$

(g) $\lim_{x \to \infty} \dfrac{2x^2 - x - 1}{x^2 - 1}$

For an infinite limit involving a fraction, only the higher order terms matter. More formally, we can divide by the highest power of x represented in the fraction.

$$\lim_{x \to \infty} \frac{2x^2 - x - 1}{x^2 - 1} = \lim_{x \to \infty} \frac{2 - \frac{1}{x} - \frac{1}{x^2}}{1 - \frac{1}{x^2}} = \frac{2}{1} = 2$$

10. Given the following graph:



(a) (3 points) Find $\lim_{x\to 1^-} f(x) = -1$

(b) (3 points) Find $\lim_{x\to 1^+} f(x) = 2$

(c) (3 points) Find $\lim_{x\to 4} f(x) = -1$

(d) (3 points) Find $\lim_{x\to \infty} f(x) = 2$

(e) (5 points) List all points where $f(x)$ is discontinuous. Explain what goes wrong at each point.

Notice that $f(x)$ is discontinuous at $x = 1, x = 3$, and $x - 4$, and is continuous everywhere else.

At $x = 1$, $f(x)$ is discontinuous since $\lim_{x\to 1} f(x)$ does not exist.

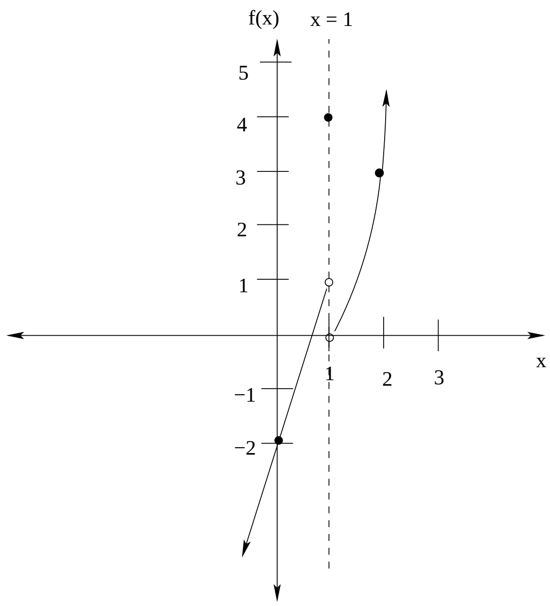
At $x = 3$, $f(x)$ is discontinuous since $f(3)$ is undefined.

At $x = 4$, $f(x)$ is discontinuous since $\lim_{x\to 4} f(x) = -1$, while $f(4) = 5$, so the limit and the function value do not agree.

11. Given the function

$$f(x) = \begin{cases} 3x-2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2-1 & \text{if } x > 1 \end{cases}$$

(a) Graph $f(x)$.



(b) Find $\lim_{x\to 1} f(x)$.

Notice that $\lim_{x\to 1^-} f(x) = 1$, while $\lim_{x\to 1^+} f(x) = 0$, so $\lim_{x\to 1} f(x)$ does not exist.

(c) Is $f(x)$ continuous at $x = 1$? Justify your answer.

No, since the function has no limit when $x = 1$, the second condition necessary for continuity at a point is violated, so the function $f(x)$ is not continuous at $x = 1$.

12. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 5x^2 - 3x - 7$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{5(x+h)^2 - 3(x+h) - 7 - (5x^2-3x-7)}{h}$$

$$= \lim_{h\to 0} \frac{5x^2 + 10xh + 5h^2 - 3x - 3h - 7 - 5x^2 + 3x + 7}{h} = \lim_{h\to 0} \frac{10xh + 5h^2 - 3h}{h} = \lim_{h\to 0} 10x + 5h - 3$$

$$= 10x - 3$$

13. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 4 - 2x - 3x^2$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{4 - 2(x+h) - 3(x+h)^2 - (4 - 2x - 3x^2)}{h}$$

$$= \lim_{h\to 0} \frac{4 - 2x - 2h - 3(x^2 + 2xh + h^2) - 4 + 2x + 3x^2}{h}$$

$$= \lim_{h\to 0} \frac{4 - 2x - 2h - 3x^2 - 6xh - 3h^2 - 4 + 2x + 3x^2}{h} = \lim_{h\to 0} \frac{-2h - 6xh - 3h^2}{h}$$

$$= \lim_{h\to 0} \frac{h(-2 - 6x - 3h)}{h} = \lim_{h\to 0} -2 - 6x - 3h = -2 - 6x.$$

14. Suppose $f(x) = x^3 - 3x^2 + 5$.

   (a) Find the equation for the tangent line to $f(x)$ when $x = 1$.

   First, we find the point of tangency by evaluating $f(x)$ when $x = 1$:

   $f(1) = 1^3 - 3(1)^2 + 5 = 1 - 3 + 5 = 3$.

   Next, we find the slope of the tangent line by evaluating the derivative of $f(x)$ when $x = 1$:

   $f'(x) = 3x^2 - 6x$, so $f'(1) = 3 - 6 = -3$.

   Finally, we use the point slope formula to find the equation for the line:

   $y - 3 = -3(x - 1) = -3x + 3$. so $y = -3x + 6$.

   (b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

   Recall that the tangent line to a function is horizontal if and only if slope of the tangent line is zero, that is, when the derivative of the function is zero.

   Therefore, we consider the equation $f'(x) = 3x^2 - 6x = 0$, or $3x(x - 2) = 0$. This equation has two solutions: $x = 0$, and $x = 2$.

15. Suppose $f(x) = (x + 1)^{\frac{3}{2}}$.

   (a) Find the equation for the tangent line to $f(x)$ when $x = 3$.

   First notice that when $x = 3$, $f(x) = (3 + 1)^{\frac{3}{2}} = 4^{\frac{3}{2}} = 2^3 = 8$, so a the point on our curve where the tangent line meets the curve is $(3, 8)$.

   Next, to find the slope of the tangent line, we find the derivative of $f(x)$ when $x = 3$. $f'(x) = \frac{3}{2}(x + 1)^{\frac{1}{2}}$, so $f'(3) = \frac{3}{2}(3 + 1)^{\frac{1}{2}} = \frac{3}{2}(2) = 3$.

   Thus, applying the point slope formula using this information, we get

   $y - 8 = 3(x - 3)$, or $y = 3x - 1$.

   (b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

   In part (a) above, we found that $f'(x) = \frac{3}{2}(x + 1)^{\frac{1}{2}}$.

   The tangent line to the original function $f(x)$ is horizontal precisely when the derivative function is zero.

   This occurs when $\frac{3}{2}(x + 1)^{\frac{1}{2}} = 0$, or when $(x + 1)^{\frac{1}{2}} = 0 \cdot \frac{2}{3} = 0$. That is, when $x + 1 = 0$.

   Therefore the only horizontal tangent line occurs when $x = -1$.

16. Find the derivative of each of the following functions. You **do not** have to use the limit definition, and you **do not** need to simplify your answers.

   (a) $h(x) = x^3 + \sqrt{x^3}$

   First, we rewrite $h(x) = x^3 + x^{\frac{3}{2}}$. Therefore, $f'(x) = 3x^2 + \frac{3}{2}x^{\frac{1}{2}}$

   (b) $f(x) = 5x^4 - 3x^2 + \frac{2}{x}$

   First notice that $f(x) = 5x^4 - 3x^2 + \frac{2}{x} = f(x) = 5x^4 - 3x^2 + 2x^{-1}$

   Using the Power Rule, $f'(x) = 20x^3 - 6x - 2x^{-2}$

   (c) $h(x) = \dfrac{5x^3 - 4x^2 + 7x}{x^2}$

   Here, we again rewrite $h(x)$ in order to obtain $h(x) = \frac{5x^3}{x^2} - \frac{4x^2}{x^2} + \frac{7x}{x^2} = 5x - 4 + 7x^{-1}$.

   Therefore, $h'(x) = 5 - 7x^{-2}$.

(d) $h(x) = (x^2 - 4x^3)(4x^3 + 3x^2 - 7x + 3)$

Using the product rule: $h'(x) = f'(x)g(x) + f(x)g'(x)$,

$h'(x) = (2x - 12x^2)(4x^3 + 3x^2 - 7x + 3) + (x^2 - 4x^3)(12x^2 + 6x - 7)$.

(e) $f(x) = (2x^2 + 5x - 4)(x^3 + 2x^2 - 1)$

Using the Product Rule, $f'(x) = (2x^2 + 5x - 4)(3x^2 + 4x) + (4x + 5)(x^3 + 2x^2 - 1)$.

(f) $f(x) = \dfrac{2x+3}{x^2-1}$

Using the Quotient Rule, $f'(x) = \dfrac{(x^2-1)(2) - (2x+3)(2x)}{(x^2-1)^2}$.

(g) $h(x) = (x^3 - 2x + 1)^{\frac{5}{2}}$

Using the Chain rule: $h'(x) = g'(f(x))f'(x)$, $h'(x) = \frac{5}{2}(x^3 - 2x + 1)^{\frac{3}{2}}(3x^2 - 2)$

(h) $f(x) = \sqrt{2x^2+1}$

First notice that $f(x) = \sqrt{2x^2+1} = (2x^2+1)^{\frac{1}{2}}$

By the Chain Rule, $f'(x) = \frac{1}{2}(2x^2+1)^{-\frac{1}{2}}(4x) = 2x(2x^2+1)^{-\frac{1}{2}}$

(i) $\left(\dfrac{2-4x^3}{x^2-1}\right)^4$

This derivative requires both the Chain rule and the quotient rule.

If we think of $h(x) = g(f(x))$, where $g(x) = x^4$, and $f(x) = \frac{2-4x^3}{x^2-1}$, then since $f'(x) = \frac{(-12x^2)(x^2-1)-(2-4x^3)(2x)}{(x^2-1)^2}$, we see that

$$h'(x) = 4\left(\frac{2-4x^3}{x^2-1}\right)^3 \cdot \frac{(-12x^2)(x^2-1) - (2-4x^3)(2x)}{(x^2-1)^2}$$

(j) $f(x) = (x^2+1)(x^3 - 2x + 1)^{\frac{3}{2}}$

Using the Product Rule and the Chain Rule,

$f'(x) = (x^2+1)\frac{3}{2}(x^3 - 2x + 1)^{\frac{1}{2}}(3x^2 - 2) + (2x)(x^3 - 2x + 1)^{\frac{3}{2}}$

17. Suppose you own a company that manufactures widgets, and the demand equation for them is given by $3x + 4p = 120$.

(a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(40)$.

To find $R(x)$, we solve the demand equation for p, yielding $4p = 120 - 3x$, or $p = 30 - \frac{3}{4}x$.

Since revenue is price times quantity, $R(x) = (30 - \frac{3}{4}x)x = 30x - \frac{3}{4}x^2$.

Therefore, $R(10) = 30(10) - \left(\frac{3}{4}\right)(10)^2 = 300 - \frac{300}{4} = 300 - 75 = \$225$.

Similarly, $R(40) = 30(40) - \left(\frac{3}{4}\right)(40)^2 = 1200 - 1200 = \$0$.

(b) Find the marginal revenue function $R'(x)$

$R'(x) = 30 - \frac{3}{2}x$.

(c) Compute $R'(10)$ and $R'(40)$ and explain what these numbers mean in practical terms.

$R'(10) = 30 - \left(\frac{3}{2}\right)(10) = 30 - 15 = 15$. This means that when 10 units have been sold, revenue is changing at \$ 15 per widget, that is, if an additional widget were sold, revenue would increase by about \$ 15.

$R'(40) = 30 - \left(\frac{3}{2}40\right) = 30 - 60 = -30$. This means that when 40 units have been sold, revenue is changing at \$-30 per widget, that is, if an additional widget were sold, revenue would decrease by about \$ 30.

(d) If $C(x) = 20x + \frac{1}{4}x^2 + 100$, find $P(x)$ and use it to compute $P(10)$.

Recall that $P(x) = R(x) - C(x) = 30x - \frac{3}{4}x^2 - (20x + \frac{1}{4}x^2 + 100) = 10x - x^2 - 100$.

Therefore, $P(10) = 10(10) - 10^2 - 100 = 100 - 100 - 100 = -100$.

(e) Find the marginal profit function $P'(x)$, use it to compute $P'(5)$, and explain what this means in practical terms.

$P'(x) = 10 - 2x$

$P'(5) = 10 - 2(5) = 0$. In practical terms, this means that when 5 widgets have been sold, profit is changing at \$0 per widget. That is, profit is not changing at this point in time.

18. Suppose you own a company that manufactures snow globes, and the demand equation for them is given by $5x+4p = 200$.

(a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(30)$.

Recall that Revenue is price times quantity sold. Solving the demand equation for p, we get $4p = 200 - 5x$, or $p = 50 - \frac{5}{4}x$. Therefore, $R(x) = p \cdot x = 50x - \frac{5}{4}x^2$.

$R(10) = 50(10) - \frac{5}{4}(10)^2 = 500 - \frac{5}{4}(100) = 500 - 125 = 375$

$R(30) = 50(30) - \frac{5}{4}(30)^2 = 1500 - \frac{5}{4}(900) = 1500 - 1125 = 375$

(b) Find the marginal revenue function $R'(x)$

$R'(x) = 50 - \frac{5}{2}x$

(c) Compute $R'(10)$ and $R'(30)$ and explain what these numbers mean in practical terms.

$R'(10) = 50 - \frac{5}{2}(10) = 50 - 25 = 25$

$R'(30) = 50 - \frac{5}{2}(30) = 50 - 75 = -25$

Notice that the marginal revenue is positive when $x = 10$ and negative when $x = 30$.

When 10 units are sold, the approximate revenue added by selling an additional unit is \$25.

When 30 units are sold, when an additional unit is sold, \$25 of revenue would be lost.