Math 229
Exam 1 - Practice Problems

1. Given the points $A : (-2, 4)$ and $B : (7, -1)$:
   (a) Find an equation for the line containing A and B
   (b) Find the equation for the line through the origin and perpendicular to $\overleftrightarrow{AB}$
2. Given the equation $4x + 3y = -2$
   (a) Find the slope of the line represented by this equation.
   (b) Find the x and y intercepts for this line.
   (c) Give an equation for a line through $(-2, 4)$ that is parallel to this line.
3. Suppose you own a company that manufactures widgets. Your supplier sells you the widgets wholesale at \$8 apiece. It costs you \$750 a month to rent your store, and you spend an additional \$2150 each month on utilities, supplies, and employee salaries. You sell the widgets at a retail price of \$15 apiece.
   (a) Find an equation $C(x)$ that gives your monthly costs, where x is the number of widgets you purchase for sale that month.
   (b) Find equations for your monthly revenue, $R(x)$, and your monthly profit, $P(x)$, assuming that you sell all of the new widgets that you purchase.
   (c) How many widgets do you need to sell each month in order to break even?
   (d) Suppose that instead of purchasing your widgets wholesale, you could instead build a factory and make your own widgets for only \$4 apiece. If the factory would cost \$750,000 to build, but all of your other costs would remain the same, how many widgets would you need to sell during the lifetime of the factory in order to make the construction worthwhile?
4. Simplify the following:
   (a) $\left(\frac{3}{4}\right)^{-2}$
   (b) $\left(\frac{y^{12}}{25z^4}\right)^{-\frac{3}{2}}$
   (c) $\sqrt[5]{32x^{11}y^{14}z^8}$
   (d) $\left(\frac{(5xyz)^2z^{-2}}{2x^{-2}y^2z^{-4}}\right)^{-1}$
   (e) $\frac{3x^2 - 10x + 3}{x^2 + x - 1} \cdot \frac{x^2 + x - 2}{x^2 - 9}$
   (f) $\frac{2x^2 + 4}{2x^2 + 7x - 4} - \frac{x - 1}{x + 4}$
   (g) $\frac{\frac{1}{x} + \frac{3}{x-2}}{\frac{4}{x-1} - \frac{2}{x-2}}$
   (h) $\frac{\frac{3}{2x+2h+1} - \frac{3}{2x+1}}{h}$
5. Rationalize the denominator in the following expressions:
   (a) $\frac{3x}{\sqrt[3]{x}}$
   (b) $\frac{2x + 3}{\sqrt{2x} - 1}$
6. Factor each of the following expressions completely:
   (a) $2x^2 + x - 6$
   (b) $50x^2 + 45x - 18$
   (c) $16x^2 - 25y^2$
   (d) $6x^3y - 27x^2y - 15xy$
   (e) $3x^3 + x^2 - 3x - 1$
7. Given the function $f(x) = \frac{2}{x - 4}$
   (a) What is the domain of f? Give your answer in interval notation.
   (b) Find $f(12)$
   (c) Find $f(2a + 4)$
   (d) Find $\frac{f(a + h) - f(a)}{h}$. Simplify your answer.
8. Given that $f(x) = \frac{1}{3x - 2}$ and $g(x) = \sqrt{x^2 - 4}$
   (a) Find $\frac{g}{f}(x)$
   (b) Find $g \circ f(x)$
   (c) Find $f \circ g(2)$

9. Evaluate the following limits. Be sure to show enough work to justify your answers.

(a) $\lim_{x \to 0} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

(b) $\lim_{x \to 2} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

(c) $\lim_{x \to 2} \dfrac{x}{x - 2}$

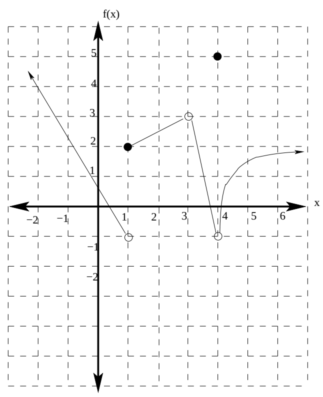
(d) $\lim_{x \to \infty} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

(e) $\lim_{x \to 0} \dfrac{2x^2 - x - 1}{x^2 - 1}$

(f) $\lim_{x \to 1} \dfrac{2x^2 - x - 1}{x^2 - 1}$

(g) $\lim_{x \to \infty} \dfrac{2x^2 - x - 1}{x^2 - 1}$

10. Given the following graph:



(a) Find $\lim_{x \to 1^-} f(x)$

(b) Find $\lim_{x \to 1^+} f(x)$

(c) Find $\lim_{x \to 4} f(x)$

(d) Find $\lim_{x \to \infty} f(x)$

(e) List all points where $f(x)$ is discontinuous. Explain what goes wrong at each point.

11. Given the function

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$$

(a) Graph $f(x)$.

(b) Find $\lim_{x \to 1} f(x)$.

(c) Is $f(x)$ continuous at $x = 1$? Justify your answer.

12. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 5x^2 - 3x - 7$

13. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 4 - 2x - 3x^2$

14. Suppose $f(x) = x^3 - 3x^2 + 5$.

(a) Find the equation for the tangent line to $f(x)$ when $x = 1$.

(b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

15. Suppose $f(x) = (x + 1)^{\frac{3}{2}}$.

(a) Find the equation for the tangent line to $f(x)$ when $x = 3$.

(b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

16. Find the derivative of each of the following functions. You **do not** have to use the limit definition, and you **do not** need to simplify your answers.

    (a) $h(x) = x^3 + \sqrt{x^3}$

    (b) $f(x) = 5x^4 - 3x^2 + \frac{2}{x}$

    (c) $h(x) = \dfrac{5x^3 - 4x^2 + 7x}{x^2}$

    (d) $h(x) = (x^2 - 4x^3)(4x^3 + 3x^2 - 7x + 3)$

    (e) $f(x) = (2x^2 + 5x - 4)(x^3 + 2x^2 - 1)$

    (f) $f(x) = \dfrac{2x + 3}{x^2 - 1}$

    (g) $h(x) = (x^3 - 2x + 1)^{\frac{5}{2}}$

    (h) $f(x) = \sqrt{2x^2 + 1}$

    (i) $\left(\dfrac{2 - 4x^3}{x^2 - 1}\right)^4$

    (j) $f(x) = (x^2 + 1)(x^3 - 2x + 1)^{\frac{3}{2}}$

17. Suppose you own a company that manufactures widgets, and the demand equation for them is given by $3x + 4p = 120$.

    (a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(40)$.

    (b) Find the marginal revenue function $R'(x)$

    (c) Compute $R'(10)$ and $R'(40)$ and explain what these numbers mean in practical terms.

    (d) If $C(x) = 20x + \frac{1}{4}x^2 + 100$, find $P(x)$ and use it to compute $P(10)$.

    (e) Find the marginal profit function $P'(x)$, use it to compute $P'(5)$, and explain what this means in practical terms.

18. Suppose you own a company that manufactures snow globes, and the demand equation for them is given by $5x+4p = 200$.

    (a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(30)$.

    (b) Find the marginal revenue function $R'(x)$

    (c) Compute $R'(10)$ and $R'(30)$ and explain what these numbers mean in practical terms.