Math 229
Exam 3 Practice Problems

1. Solve the following equations (give exact answers whenever possible):
   (a) $e^{3x-2} = e^{4-5x}$
   (b) $9^{2x} = 27(3)^{2x+1}$
   (c) $\log_2(3x^2 - 3) = \log_2(x^2 + x)$
   (d) $\log_5(x^2 + 21) = 2$
   (e) $\log_2(2x) + \log_2(x - 3) = 3$
   (f) $e^{2x-1} = 3$

2. Determine whether the following are True or False:
   (a) $\ln\left(\dfrac{x^3}{(x+1)(x-1)}\right) = 3\ln x - \ln(x+1) + \ln(x-1)$
   (b) $e^{\ln(x^2+1)} = x^2 + 1$
   (c) $e^{x^2} \cdot e^{3x} = e^{3x^3}$
   (d) $\dfrac{\ln(4x)}{\ln(2x)} = \ln 2$
   (e) $\ln\left(e^{x^2} - 4\right) = x^2 - \ln 4$

3. Find the exact value of the following logarithmic expressions:
   (a) $\log_2(32)$
   (b) $\log_3\left(\frac{1}{27}\right)$
   (c) $\log_5(1)$
   (d) $\log_4 32$

4. Use the laws of logarithms to simplify the expression: $\ln\left(\dfrac{x^2(x-1)^{\frac{5}{2}}}{(x-4)^3}\right)$

5. Use the change of base formula to approximate the following:
   (a) $\log_5 10$
   (b) $\log_9 12$
   (c) $\log_{15} 7$

6. (a) Suppose you invest \$10,000 in a savings account that pays 3% annual interest compounded monthly. How much money will be in the account after 6 years?
   (b) How long would it take \$5,000 invested at 6% annual interest compounded continuously to triple?
   (c) Find the interest rate needed for an investment of \$2,000 to double in 6 years if the interest is comounded quarterly.

7. Suppose that a culture of bacteria that initially has 500 cells grows to 10,000 cells in 12 hours.
   (a) Find a function $f(t)$ that gives the number of cells in the culture as a function of time (in hours), assuming that this population grows continuously and exponentially.
   (b) How long will it take for the culture to reach 1,000,000 cells?

8. Compute the derivatives of the following functions. You do not need to simplify your answers.
   (a) $f(x) = e^{3x^2}$
   (b) $g(x) = \ln(3x^2 - 4x + 6)$
   (c) $h(x) = (x^2 + 1)e^{x^2+1}$
   (d) $k(x) = x^2 \ln(e^x + 1)$
   (e) $f(x) = \ln\left(\dfrac{x^2}{(2x-1)^3}\right)$
   (f) $g(x) = e^{e^{2x^3}}$
   (g) $h(x) = \ln(x^2 + 1)e^{x^3}$

9. Find the tangent line to $f(x) = x\ln(2x)$ when $x = \frac{1}{2}$

10. Find the tangent line to $f(x) = 2xe^{2x-4}$ when $x = 2$

11. Determine the intervals where the function $g(t) = t^4e^{2t}$ in increasing and the intervals where it is decreasing.

12. Find the absolute extrema of $g(t) = t^2e^{2t}$ on the interval $[-2, 2]$.

13. Suppose $\int_0^2 f(x)\,dx = 4$ and $\int_0^2 g(x)\,dx = 2$. Find $\int_0^2 2f(x) - g(x)\,dx$

14. Evaluate the following integrals:

(a) $\int 6x^3 - 4x^{\frac{1}{2}}\,dx$

(b) $\int 5x^4 - x^{\frac{3}{2}}\,dx$

(c) $\int \frac{4x^3 - 3x^2 + 2x}{2x^2}\,dx$

(d) $\int_{-1}^{1} 3x^5 - 4x^3\,dx$

(e) $\int_0^3 4x^2 - e^{3x}\,dx$

(f) $\int_0^4 e^{3x} + x^{-\frac{1}{2}}\,dx$

(g) $\int 5x(x^2+1)^8\,dx$

(h) $\int_0^1 x^2(2x^3+1)^2\,dx$

15. Assume f is continuous on $[-5, 3]$, $\int_{-5}^{-1} f(x)\,dx = -7$, $\int_{-1}^{3} f(x)\,dx = 4$, and $\int_1^3 f(x)\,dx = 2$. Find:

(a) $\int_3^{-1} f(x)\,dx$

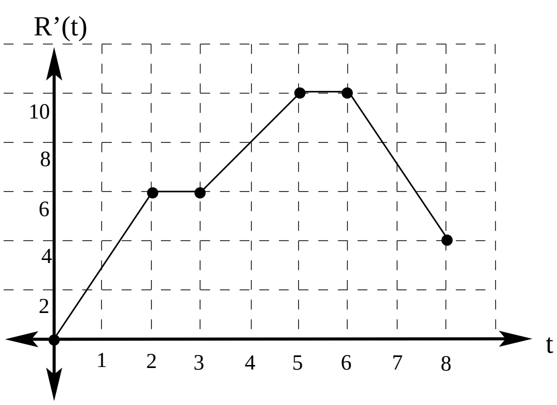
(b) $\int_{-5}^{1} f(x)\,dx$

(c) $\int_{-5}^{3} f(x)\,dx$

(d) $\int_{-1}^{-1} f(x)\,dx$

(e) Find the average value of f on $[-5, -1]$

16. Suppose marginal revenue, $R'(t)$ is given by the graph below, where t in in months and $R'(t)$ is in \$1000s of dollars per month. Find the total revenue from $t = 0$ to $t = 6$.



17. Find the average value of $f(x) = x^2 - \frac{1}{x^2}$ for $1 \le x \le 3$.

18. Find the area of the region enclosed by the graphs $f(x) = x$ and $g(x) = \sqrt{x}$.

19. Find the area of the region enclosed by the graphs $f(x) = x^2 - 4$ and $g(x) = 4 - x^2$.