Math 229: Final Exam Review Sheet

Final Exam: Friday, May 9th 11:30am-1:30pm, Bridges Room 268

Part 1: Algebra Review Material

- Plotting points, lines, slope, x and y-intercepts, parallel and perpendicular lines
- Functions, the Vertical Line Test, Cost, Revenue, and Profit functions, demand equations
- Solving linear equations using Substitution and Elimination, related applications.
- Solving Inequalities, Factor Analysis, Interval Notation
- The Domain and Range of a Function, Evaluating Functions, The sum, difference, product, quotient, and composition of functions.

Key Formulas:
$m = \frac{y_2 - y_1}{x_2 - x_1}$, $y - y_1 = m(x - x_1)$, $y = mx + b$
$R(x) = x \cdot p(x)$, $P(x) = R(x) - C(x)$

Not Tested: absolute value equations and inequalities.

Part 2: Limits, Continuity, and Computing Derivatives

- Definition of a limit, evaluating limits, showing a limit does not exist, limits at infinity
- One sided limits (definition and evaluating them), continuity, finding points of discontinuity, piecewise defined functions
- The limit definition of the derivative, finding derivatives using limits (the 4 step process), finding the equation of a tangent line
- Differentiation rules [sums, products, quotients, and the chain rule], higher order derivatives
- Marginal functions [$C'(x), R'(x), P'(x)$, average cost $\overline{C}(x)$].

Key Formulas:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$\frac{d}{dx} x^r = r x^{r-1}$$
$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$
$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$$
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$
$$\frac{d}{dx}\left(g(f(x))\right) = g'(f(x))f'(x)$$

Not Tested: Elasticity of Demand, the Intermediate Value Theorem

Part 3: Applications of the Derivative and Curve Sketching

- Applications of the first derivative (increasing/decreasing, critical points, relative extrema)
- Applications of the second derivative (concavity, inflection points, the second derivative test)
- Vertical and Horizontal Asymptotes, intercepts, symmetry, curve sketching.
- Absolute extrema and Optimization problems.

Key Concepts:
A function is increasing when $f'(x) > 0$, decreasing when $f'(x) < 0$, critical numbers are where $f'(x) = 0$ or is undefined.
A function is concave up when $f''(x) > 0$, concave down when $f''(x) < 0$. Inflection points are when the function changes concavity.
Relative Extrema occur at critical points, absolute extrema occur either at critical points or at the end points of the interval under consideration.

Part 4: Exponentials and Logarithms

- Exponential Functions (definition, properties, graphs)
- Logarithmic Functions (definition, properties, graphs)
- Compound interest, continuous interest and the base e, simplifying log expressions
- Differentiating exponential and logarithmic functions

Key Formulas:
$\log_b(x) = y$ if and only if $b^y = x$
$\log(xy) = \log(x) + \log(y)$, $\log(\frac{x}{y}) = \log(x) - \log(y)$, and $\log(x^a) = a\log(x)$
$A = P(1 + \frac{r}{n})^{nt}$ and $A = Pe^{rt}$
$\frac{d}{dx}(\ln(f(x))) = \frac{f'(x)}{f(x)}$, and $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$
$\int e^x \; dx = e^x + C$
$\int \frac{1}{x} \; dx = \ln|x| + C.$

Part 5: Antidifferentiation and the Fundamental Theorem of Calculus
- Antiderivatives, initial value problems, indefinite integrals
- Approximating Definite integrals by using rectangles.
- The Fundamental Theorem of Calculus, using the FTC to evaluate definite integrals.
- Using Integration by substitution for both definite and indefinite integrals.
- Finding the area under a curve, computing the average value of a function, finding the area between two curves.

Key Formulas:
$\int a \; dx = ax + C$ for any constant a
$\int x^r \; dx = \frac{1}{r+1}x^{r+1} + C$

The Fundamental Theorem of Calculus:
Let f be a continuous function on the interval $[a, b]$, and let F be an antiderivative of f.
Then $\int_a^b f(x) \; dx = F(b) - F(a)$

$\int_a^b f(x) \; dx = -\int_b^a f(x) \; dx$
$\int_a^b cf(x) \; dx = c\int_a^b f(x) \; dx$ for any constant c
$\int_a^b f(x) \pm g(x) \; dx = \int_a^b f(x) \; dx \pm \int_a^b g(x) \; dx$
$\int_a^b f(x) \; dx = \int_a^c f(x) \; dx + \int_c^b f(x) \; dx$ for any $a < c < b$
Let f be a function that is integrable on an interval $[a, b]$. Then the **average value** of f over $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x) \; dx$.
Given two functions f and g, with $g(x) \geq f(x)$ on an interval $[a, b]$, the area between f and g between a and b is given by $\int_a^b g(x) - f(x) \; dx$.

Part 6: Matrices

- Representing a system of linear equations using a matrix.
- Solving systems using matrix techniques.
- Determining whether a system has a unique solutions versus infinitely many solutions or no solutions.
- Operations on Matrices: equality, addition/subtraction, scalar multiplication, multiplying matrices
- Finding the inverse of a matrix, solving systems of equations using the inverse of a matrix