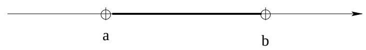
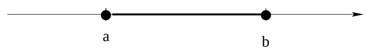
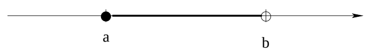
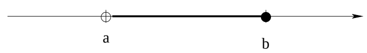
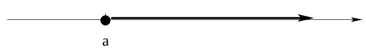
Math 229
Inequalities and Absolute Value

A. Interval Notation

Finite Intervals:

Interval	Inequality	Graph
Open: (a,b)	$a < x < b$	
Closed: $[a,b]$	$a \leq x \leq b$	
Half Open: $[a,b)$	$a \leq x < b$	
$(a,b]$	$a < x \leq b$	

Infinite Intervals:

Interval	Inequality	Graph
(a,∞)	$a < x$	
$[a,\infty]$	$a \leq x$	
$(-\infty,b)$	$x < b$	
$(-\infty,b]$	$x \leq b$	

B. Properties of Inequalities

1. If $a < b$ and $b < c$ then $a < c$ [Transitivity]
2. If $a < b$, then $a + c < b + c$ [Additive Shift]
3. If $a < b$ and $c > 0$, then $ac < bc$ [Positive Multiplication]
4. If $a < b$ and $c < 0$, then $ac > bc$ [Negative Multiplication - Inequality Reversal!]

C. Solving Inequalities

Our goal is to use the properties of inequalities and other algebraic techniques to find which real numbers satisfy a given inequality.

Examples:

1. $3 - 2x < 5$

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  3 - 2x <  5
 -3         -3
    - 2x < 2
      \-2  \-2
       x > -1
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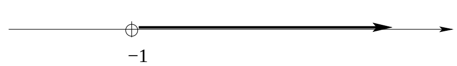
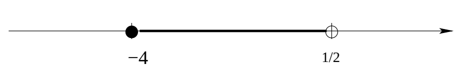


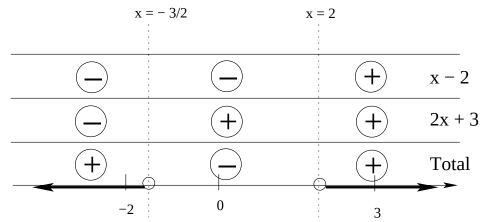
2. $3 < 4 - 2x \leq 12$

$3 < 4 - 2x \leq 12$
$-4 \;\; -4 \qquad\quad -4$
$-1 < -2x \quad \leq 8$
$\div -2 \;\; \div -2 \quad \div -2$
$\frac{1}{2} > \;\; x \quad \geq -4$



3. $2x^2 - x - 6 > 0$ – for this one, we will need to use "sign analysis"

$(2x+3)(x-2) > 0$
Solving the related linear equations:
$2x + 3 = 0 \rightarrow x = -\frac{3}{2}$
$x - 2 = 0 \rightarrow x = 2$



Therefore, the solution to this inequality is: $(-\infty, -\frac{3}{2}) \cup (2, \infty)$

D. Absolute Value
Definition:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Examples: (a) $|7| = 7$ (b) $|-4| = 4$

Properties:

1. $|-a| = |a|$
2. $|ab| = |a||b|$
3. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
4. $|a + b| \leq |a| + |b|$

Evaluating Absolute Value Expressions:
$(i)|\pi - 3| = \pi - 3$ $(ii)|3 - \pi| = \pi - 3$

$(iii)\dfrac{|-4| - |7|}{|-4-7|} = \dfrac{4-7}{11} = \dfrac{-3}{11}$

Absolute Value Equations and Inequalities:
Note: Since absolute value acts differently on positive numbers than it does on negative numbers, we will need to look at positive and negative cases in order to solve equations and inequalities involving absolute value.

Examples:

1. $|x + 3| = 4$

positive case:
$x + 3 = 4$
$-3 \quad -3$
$x = 1$

negative case:
$-(x + 3) = 4$
or
$x + 3 = -4$
$-3 \quad -3$
$x = -7$

2. $|2x - 3| \leq 7$

positive case:
$2x - 3 \leq 7$
$+3 \quad +3$
$2x \leq 10$
so $x \leq 5$

negative case:
$-(2x - 3) \leq 7$
or
$2x - 3 \geq -7$
$+3 \quad +3$
$2x \geq -4$
so $x \geq -2$

Thus $-2 \leq x \leq 5$

3. $|3x + 2| < -4$
Solution: ??