Math 229
Limits and Continuity Handout

**A. Limits:** The function $f$ has **limit** $L$ as $x$ approaches $a$, written $\lim_{x\to a} f(x) = L$ if the value if $f(x)$ can be made as close to the number $L$ as we please by taking $x$ sufficiently close to (but not equal to) $a$.

**Example:** $\lim_{x\to 1} 5x - 2 = 3$

**B. Properties of Limits:** Suppose $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$. Then:

1. $\lim_{x\to a} [f(x)]^r = [\lim_{x\to a} f(x)]^r = L^r$
2. $\lim_{x\to a} c \cdot f(x) = c \cdot \lim_{x\to a} f(x) = c \cdot L$
3. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = L \pm M$
4. $\lim_{x\to a} [f(x)g(x)] = [\lim_{x\to a} f(x)] \cdot [\lim_{x\to a} g(x)] = L \cdot M$
5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \dfrac{L}{M}$

**C. Limits at Infinity:** The function $f$ has **limit** $L$ as $x$ increases without bound (or as $x$ approaches infinity), written $\lim_{x\to\infty} f(x) = L$ if the value if $f(x)$ can be made as close to the number $L$ as we please by taking $x$ to be sufficiently large. Similarly, the function $f$ has **limit** $L$ as $x$ decreases without bound (or as $x$ approaches negative infinity), written $\lim_{x\to -\infty} f(x) = M$ if the value if $f(x)$ can be made as close to the number $M$ as we please by taking $x$ to be sufficiently large in the negative direction.

**Example:** $\lim_{x\to\infty} \dfrac{3x+1}{x-10} = 3$

**D. One Sided Limits:** The function $f$ has **right-hand limit** $L$ as $x$ approaches $a$ from the right, written $\lim_{x\to a^+} f(x) = L$ if the value if $f(x)$ can be made as close to the number $L$ as we please by taking $x$ to be sufficiently close to $a$ but greater than $a$. Similarly, the function $f$ has **left-hand limit** $L$ as $x$ approaches $a$ from the left, written $\lim_{x\to a^-} f(x) = L$ if the value if $f(x)$ can be made as close to the number $L$ as we please by taking $x$ to be sufficiently close to $a$ but less than $a$.

**E. Continuity:** A function $f$ is **continuous at the point** $x = a$ if the following hold:

1. $f(a)$ is defined
2. $\lim_{x\to a} f(x)$ exists (that is both its one-sided limits exist and agree)
3. $\lim_{x\to a} f(x) = f(a)$

**Notes:**

1. Constant functions and polynomial functions are continuous everywhere.
2. Rational functions $R(x) = \frac{f(x)}{g(x)}$, with $f$ and $g$ polynomials are continuous except where $g(x) = 0$.
3. If two functions $f$ and $g$ are continuous at a point $a$, then so are: $f \pm g$, $fg$, and $\frac{f}{g}$ (provided $g(a) \neq 0$.