Math 262
Exam 2 - Practice Problems

1. Determine whether or not each of the following functions is one-to-one.

   (a) $f(x) = \ln(x^2)$
   (b) $f(x) = x^5 + 2x^3 - 2$

2. Determine whether or not each of the following functions is one-to-one. If it is, find the inverse function.

   (a) $f(x) = x^3 + 4$
   (b) $f(x) = x^4 - 2$
   (c) $f(x) = \dfrac{x+3}{2x-1}$

3. (4 points each) Given the following table of values and that $g(x) = f^{-1}(x)$:

	$f(x)$	$f'(x)$
0	4	-1
1	2	3
2	1	5
3	-5	7
4	3	-2

   Find the following:
   (a) $g(2)$ (b) $g(4)$ (c) $g'(4)$ (d) $g'(3)$

4. *Without* solving for the inverse function $f^{-1}(x)$, find the derivative of the inverse function at the given x value:

   (a) $f(x) = x^3 + 2x + 1$, $x = 1$.
   (b) $f(x) = \sqrt{x^3 + 2x + 1}$, $x = 2$

5. Find the derivative of each of the following functions:

   (a) $f(x) = \ln(\sec x + \tan x)$
   (b) $f(x) = \ln\sqrt{\dfrac{x^3}{x^5+1}}$
   (c) $f(x) = (x^5 - 4x^3 - 7)^8(1 - x^2)^7(5x + 10)^{12}$
   (d) $f(x) = e^{3x^2}\sec^{-1}(2x^3)$
   (e) $f(x) = e^{\ln(\cos(\sqrt{x}))}$
   (f) $f(x) = x3^{-x}\log_5(7x^2)$

6. Find all extrema and inflection points of the function $f(x) = x^2e^{-x}$

7. Find an equation to the tangent line to the graph of $ye^{2x} + x\ln|y| = 1 + \tan^{-1}(x)$ at the point $(0, 1)$.

8. Suppose that the population of a bacterial colony initially has 400 cells. After an hour, the population has increased to 800 cells. Find an equation for the population at any time. Then determine the population of the colony after 10 hours.

9. Scientists trying to date the age of a fossil estimate that it contains 20% of the carbon-14 originally present. Given that the half-life of carbon-14 is 5730 years, approximately how old is the fossil?

10. According to Newton's Law of Cooling, the temperature of an object placed in a room with ambient temperature T_a satisfies the differential equation $y'(t) = k[y(t) - T_a]$.

    (a) Use separation of variables to find a general solution to this differential equation.

    (b) Suppose a bowl of porridge initially at 200°F (too hot) is placed in a 70°F room. One minute later, the porridge has cooled down to 180°F. How long will it take for the porridge to cool down to 120°F (just right)?

11. Find each limit, (if it exists).

    (a) $\lim_{x \to 1} \frac{\sin(\pi x)}{x - 1}$

    (b) $\lim_{x \to 1} \frac{e^{x-1} - 1}{x^2 - 1}$

    (c) $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$

    (d) $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

    (e) $\lim_{x \to 0} \frac{x \sin x}{\cos x - 1}$

    (f) $\lim_{x \to \infty} \left(\sqrt{x^2 + 1} - x\right)$

    (g) $\lim_{x \to 0} \frac{\sin x}{\cos x}$

    (h) $\lim_{x \to \infty} \left(\frac{1}{x}\right)^{\frac{1}{x}}$

    (i) $\lim_{x \to 0^+} (\cos x)^{\frac{1}{x}}$

12. Compute each of the following *exactly*.

    (a) $\arctan(-1)$

    (b) $\arcsin(-\frac{\sqrt{3}}{2})$

    (c) $\arccos -\frac{\sqrt{2}}{2}$

13. Simplify each of the following expressions:

    (a) $\tan(\cos^{-1}(\frac{3}{5}))$

    (b) $\cot(\sin^{-1}(3x))$

14. Find all solutions to the equation $10\cos^2 x - \cos x = 3$ on the interval $[0, 2\pi)$

15. Evaluate the following integrals:

    (a) $\int \frac{3x^2}{1 + x^6}\, dx$

    (b) $\int \frac{4x}{\sqrt{1 - x^4}}\, dx$

    (c) $\int_{\sqrt{2}}^{2} \frac{4}{x\sqrt{x^2 - 1}}\, dx$

    (d) $\int_3^4 x\sqrt{x - 3}\, dx$

16. Evaluate the following integrals:

    (a) $\int \frac{1}{1 + x^2}\, dx$

    (b) $\int \frac{x}{1 + x^2}\, dx$

    (c) $\int \frac{x^2}{1 + x^2}\, dx$

    (d) $\int \frac{x^3}{1 + x^2}\, dx$