Math 262
Exam 5 - Practice Problems

1. Use a power series to approximate each of the following to within 3 decimal places:

   (a) $\arctan \dfrac{1}{2}$

   (b) $\ln(1.01)$

   (c) $\sin\left(\frac{\pi}{10}\right)$

2. For each of the following functions, find the Taylor Series about the indicated center and also determine the interval of convergence for the series.

   (a) $f(x) = e^{x-1}$, $c = 1$

   (b) $f(x) = \cos x$, $c = \dfrac{\pi}{2}$

   (c) $f(x) = \dfrac{1}{x}$, $c = -1$

3. For each of the following functions, find the Taylor Polynomial for the function at the indicated center c. Also find the Remainder term.

   (a) $f(x) = \sqrt{x}$, $c = 1$, $n = 3$.

   (b) $f(x) = \ln x$, $c = 1$, $n = 4$.

   (c) $f(x) = \sqrt{1 + x^2}$, $c = 0$, $n = 4$.

4. Estimate each of the following using a Taylor Polynomial of degree 4. Also find the error or your approximation. Finally, find the number of terms needed to guarantee an accuracy or at least 5 decimal places.

   (a) $e^{0.1}$

   (b) $\ln 0.9$

   (c) $\sqrt{1.2}$

5. Find the first four terms of the binomial series for each of the following.

   (a) $(1 + x)^{\frac{5}{4}}$

   (b) $(1 + 4x^3)^{\frac{1}{3}}$

   (c) $\dfrac{x^2}{(1 - x^2)^{\frac{1}{2}}}$

6. Use the binomial series to expand each of the following.

   (a) $(1 + x)^5$

   (b) $(1 - 4x)^4$

   (c) $(1 - 2x^5)^3$

7. Use a series to approximate $\displaystyle\int_0^1 \frac{1 - \cos x}{x^2}\, dx$ to within 5 decimal places of accuracy.

8. Use series to evaluate the limit $\displaystyle\lim_{x \to \infty} \frac{e^x - e^{-x}}{x}$

9. Express the following polar equations in rectangular coordinates:

   (a) $r = -5\cos\theta$

   (b) $r = \sin(2\theta)$

10. Express the following rectangular equations in polar coordinates:

    (a) $xy = 1$

    (b) $x^2 - y^2 = 1$

11. Find the equation for a circle with center $(0, -4)$ and passing through the origin in both rectangular and polar coordinates.

12. Graph each of the following polar equations:

    (a) $r = 1 - \sin\theta$

    (b) $r = 4 + 2\cos\theta$

    (c) $r = 3\cos(3\theta)$

    (d) $r = 2\sin(5\theta)$

    (e) $r = 3\theta$

13. Find the area of each of the following polar regions:

    (a) the region bounded by the polar graph $r = 1 + \cos\theta$

    (b) the region bounded by one loop of the curve $r = 2\sin(5\theta)$

    (c) the region inside $r = 3 + 2\sin\theta$ and outside $r = 4$

    (d) the region inside both $r = 2\cos\theta$ and $r = 2\sin\theta$