Math 262
Polar Graphs

Polar Conversion Formulas:
$x = r\cos\theta$ and $y = r\sin\theta$
$r = \sqrt{x^2 + y^2}$, or $r^2 = x^2 + y^2$, and $\tan\theta = \frac{y}{x}$ if $x \neq 0$

These conversion formulas allow us to translate both descriptions of points and equations from rectangular to polar coordinates and vice versa.

Example:

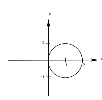
(a) To express $x^2 - y^2 = 1$ in polar coordinates, we substitute using $x = r\cos\theta$ and $y = r\sin\theta$, yielding $(r\cos\theta)^2 - (r\sin\theta)^2 = 1$, or $r^2\cos^2\theta - r^2\sin^2\theta = 1$.
Therefore, we have $r^2(\cos^2\theta - \sin^2\theta) = 1$ which, substituting using the using the double angle formula for cos gives $r^2(\cos(2\theta)) = 1$. Thus $r^2 = \frac{1}{\cos(2\theta)}$, or $r = \pm\sqrt{\frac{1}{\cos(2\theta)}}$

Graphing Polar Functions:

To graph polar functions, when possible, we write the polar function in the form $r = f(\theta)$. We then keep track of how increasing θ impacts the magnitude and direction of r. We think of gradually rotating through θ degrees counter-clockwise, and smoothly adjusting r as we go.

Example: $r = 2\cos\theta$

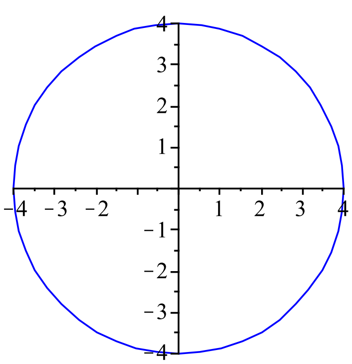
θ	$\cos\theta$	$r = 2\cos\theta$
0	1	2
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3} \approx 1.7$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\sqrt{2} \approx 1.4$
$\frac{\pi}{3}$	$\frac{1}{2}$	1
$\frac{\pi}{2}$	0	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$	-1
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\sqrt{2} \approx -1.4$
$\frac{5\pi}{6}$	$\frac{-\sqrt{3}}{2}$	$-\sqrt{3} \approx -1.7$
π	-1	-2



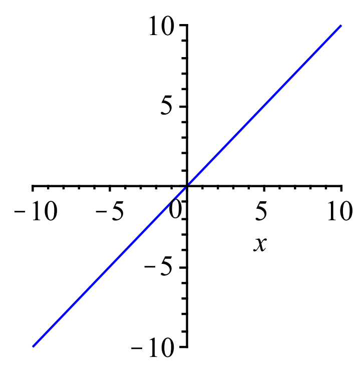
Common Polar Graphs:

1. Constant Graphs

$r = a$ (a circle of radius a centered at the origin)
Example: $r = 4$

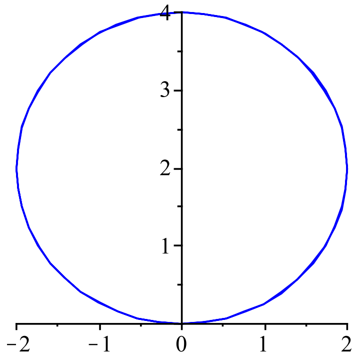


$\theta = a$ (a ray through the origin)
Example: $\theta = \frac{\pi}{4}$

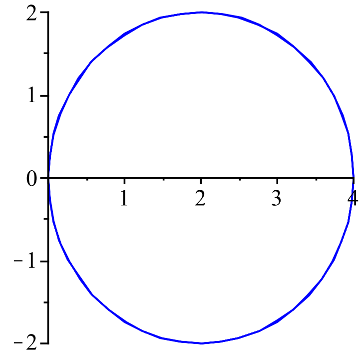


2. Circles on the coordinate axes

$r = a\sin\theta$ (a circle of radius $\frac{a}{2}$ centered at $(0, \frac{a}{2})$)
Example: $r = 4\sin\theta$

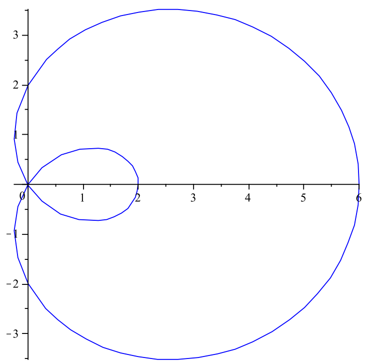


$r = a\cos\theta$ (a circle of radius $\frac{a}{2}$ centered at $(\frac{a}{2}, 0)$)
Example: $r = 4\cos\theta$

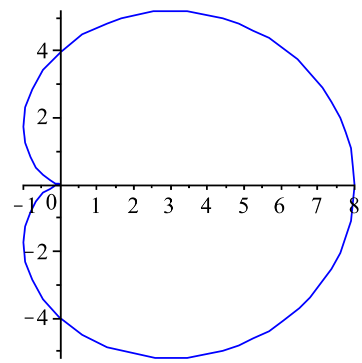


3. Graphs of the form $r = a \pm b\sin\theta$ or $r = a \pm b\cos\theta$

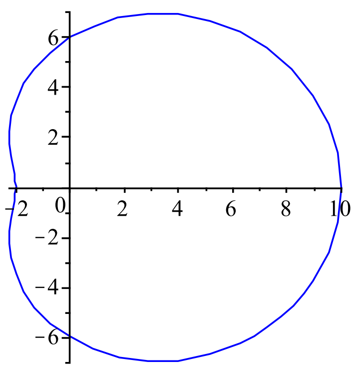
(a) $\frac{a}{b} < 1$ (a **limacon** with inner loop)
Example: $r = 2 + 4\cos\theta$



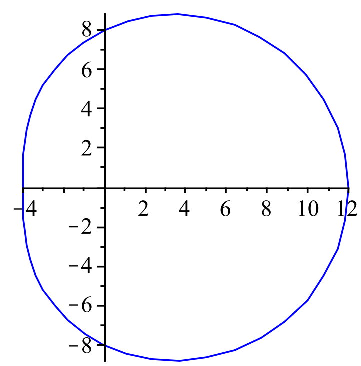
(b) $\frac{a}{b} = 1$ (a **cardioid**)
Example: $r = 4 + 4\cos\theta$



(c) $1 < \frac{a}{b} < 2$ (a **limacon** with a dimple)
Example: $r = 6 + 4\cos\theta$

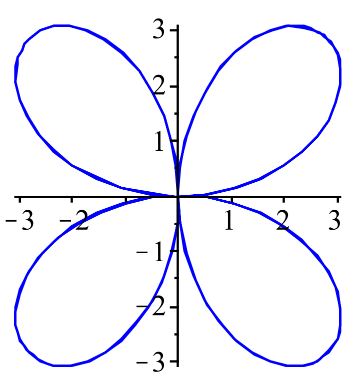


(d) $\frac{a}{b} \geq 2$ (a **convex limacon**)
Example: $r = 8 + 4\cos\theta$



4. Graphs of the form $r = a\sin n\theta$ or $r = a\cos n\theta$

(a) n even (a $2n$-leafed rose)
Example: $r = 4\sin 2\theta$



(b) n odd (an n-leafed rose)
Example: $r = 2\cos 3\theta$