1. Find the value of the following Boolean expressions:

   (a) $1 \cdot \overline{(1+0)} + \overline{0}(1+\overline{0})$

   (b) $\overline{[\overline{1} + (\overline{0} \cdot 1)]} + [\overline{0} + \overline{0} \cdot 1]$

2. Build a value table for the following Boolean functions:

   (a) $F(x,y) = x + \overline{x}y$

   (b) $F(x,y,z) = xyz + y(\overline{x} + \overline{z})$

3. Use value tables to determine whether of not the following pairs of Boolean Expressions are equivalent:

   (a) $\overline{x} + \overline{y}$ and $\overline{xy}$

   (b) $\overline{xyz}$ and $\overline{x} + \overline{y} + \overline{z}$

4. Use a 2-column proof to prove each of the following:

   (a) $(xyz) + (yz) = yz$

   (b) $\overline{(x+z) \cdot (\overline{y} + z)} = (\overline{x} + y) \cdot \overline{z}$

5. Given the following value table:

x	y	z	$F(x,y,z)$	$G(x,y,z)$	$H(x,y,z)$
0	0	0	0	1	0
0	0	1	0	0	1
0	1	0	1	0	1
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	0	0	1

   (a) Find the sum of products expansion for $F(x,y,z)$

   (b) Find the sum of products expansion for $G(x,y,z)$

   (c) Find the sum of products expansion for $H(x,y,z)$

   (d) Construct a circuit whose output is equivalent to $F(x,y,z)$

   (e) Construct a circuit whose output is equivalent to $G(x,y,z)$

   (f) Construct a circuit whose output is equivalent to $H(x,y,z)$

6. Find the sum of products expansion for each of the following Boolean Functions:

   (a) $F(x,y,z) = x + \overline{y} + x\overline{z}$

   (b) $F(x,y,z) = (x + \overline{y})z + x(\overline{y} + z)$

   (c) $F(x,y,z) = xy + \overline{x}y + \overline{y}$

   (d) $F(w,x,y,z) = (x+y)(z+\overline{w})$

7. Find the sum of products expansion of a Boolean function $F(s,t,x,y,z)$ that has value 1 if and only if an even number of the variables have value 1.

8. Given $F(x,y,z) = x(\overline{y} + z)$

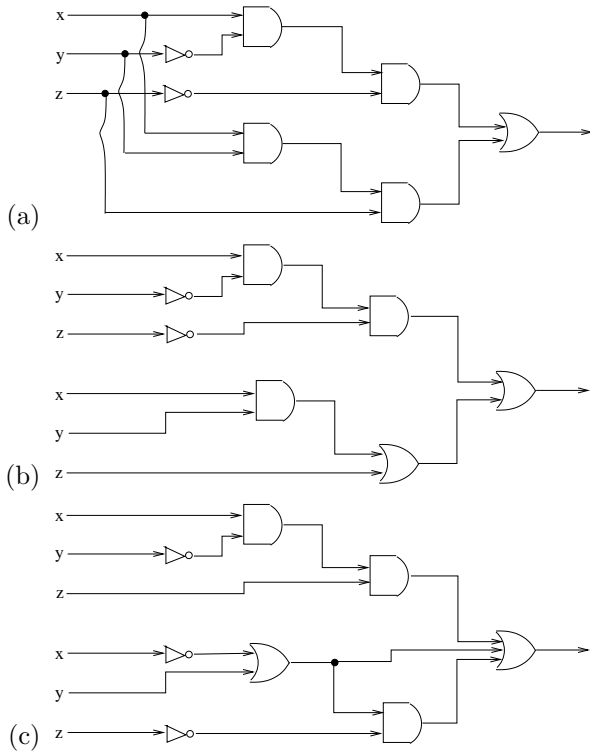   (a) Express $F(x,y,z)$ as a Boolean expression using only the operations $\cdot$ and $\overline{}$.

   (b) Express $F(x,y,z)$ as a Boolean expression using only the operations $+$ and $\overline{}$.

   (c) Express $F(x,y,z)$ as a Boolean expression using only the operation $|$.

   (d) Express $F(x,y,z)$ as a Boolean expression using only the operation $\downarrow$.

9. Find the output of each of the following circuits:

(a)

(b)

(c)

10. Recall that in the game "Paper, Scissors, Rock", two players decide which of the three symbols to show. Rock beats scissors, scissors beats paper, and paper beats rock, while if both players show the same symbol, the game ends in a tie. Suppose we decide to model this game using a circuit accepting a 6 bit input and giving a 3 bit output. The input string is in the form $p_1 s_1 r_1 p_2 s_2 r_2$ and the output is in the form $w_1 w_2 t$. For example, the input 001100 corresponds to player 1 showing rock, player 2 showing paper, so it should lead to the output 010, which corresponds to player 2 winning. Build a circuit that models this game. You can filter out illegal inputs by declaring their output to be a tie.

11. Use a K-map to minimize each of the following expressions.

   (a) $xy + \overline{x}y + \overline{x}\,\overline{y}$
   (b) $xyz + xy\overline{z} + \overline{x}yz + x\overline{y}\,\overline{z} + \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$
   (c) $(x + y)\overline{z} + x(\overline{y} + z)$
   (d) $w\overline{x}y\overline{z} + \overline{w}xyz + \overline{w}x\overline{y}\,\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$
   (e) $w\overline{x}yz + wx\,\overline{y}\,\overline{z} + \overline{w}xyz + \overline{w}x\overline{y}z + \overline{w}\,\overline{x}yz + \overline{w}\,\overline{x}y\overline{z} + \overline{w}\,\overline{x}\,\overline{y}z + \overline{w}x\,\overline{y}\,\overline{z}$

12. Given the following circuit:
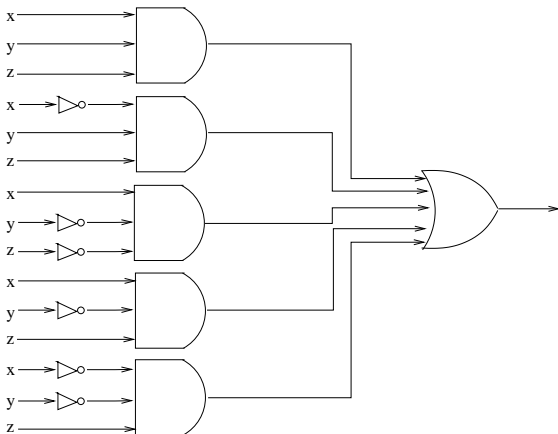
   (a) Find an expression for the output of this circuit.
   (b) Put the expression for this circuit into sum of products form.
   (c) Use K-map to minimize the expression.
   (d) Draw a circuit representing the minimized expression.
   (e) How does the efficiency of the two circuits compare?