Math 310
Exam 1 - Practice Problems

1. Translate the following English statements into symbolic form (use propositional sentences).
   (a) I will not pass this class unless I go to class every day and do all of the homework exercises.
   (b) I lock the doors and close the windows whenever I leave to go to work.
   (c) Getting up on time and getting ready quickly is sufficient for arriving at work on time.
   (d) Practicing an hour a day and getting private lessons twice a week is necessary for playing in the wind ensemble.
2. Use truth tables to determine whether or not the following pairs of statements are logically equivalent.
   (a) $[(p \wedge q) \rightarrow r]$ and $(p \rightarrow r) \wedge (q \rightarrow r)$
   (b) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$
   (c) $p \wedge (q \vee \neg r)$ and $(p \wedge q) \vee \neg(\neg p \vee r)$
3. Use truth tables to determine which of the following statements are tautologies:
   (a) $(p \oplus q) \leftrightarrow [(p \vee q) \wedge \neg(p \wedge q)]$
   (b) $(p \rightarrow q) \leftrightarrow [\neg(p \wedge \neg q)]$
   (c) $p \rightarrow (\neg q \vee r \vee \neg r)$
4. Use propositional rules of inference to prove the the following statements are tautologies:
   (a) $[\neg p \wedge (p \vee q)] \rightarrow q$
   (b) $\neg p \rightarrow (p \rightarrow q)$
5. Use propositional rules of inference to show that each pair of logical statements are logically equivalent.
   (a) $(r \vee p) \rightarrow (r \vee q)$ and $r \vee (p \rightarrow q)$.
   (b) $p \rightarrow q$ and $[(p \wedge \neg q) \rightarrow \neg p]$
6. Determine whether or not the following statements are satisfiable.
   (a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee r)$
   (b) $(p \vee q \vee \neg r \vee s) \wedge (p \vee \neg q \vee r \vee \neg s) \wedge (\neg p \vee \neg q \vee r \vee \neg s)$
7. Suppose that there is a certain town in which all the men either shave themselves, or they are shaved by the town barber Figaro (who is male). Suppose Vinny is too cheap to have the barber shave him. Let $S(x, y)$ be the two variable predicate "person x is shaved by person "y". First translate the given statement into English, then determine the truth value of the statement and justify your answer.
   (a) $\forall x\,(S(x, \text{Figaro}) \vee S(x, x))$
   (b) $\forall x\,(\neg S(x, x) \rightarrow S(x, \text{Figaro}))$
   (c) $\forall x\,(S(x, \text{Figaro}) \rightarrow \neg S(x, x))$
   (d) $\exists y \forall x\,(S(x, y))$
   (e) $\exists! x\,(S(x, x) \wedge S(x, \text{Figaro}))$
8. Translate each of the following statements into quantified predicate form. Make sure to define each predicate used and state the domain of each variable.
   (a) At least one person in this neighborhood watches television on Monday but not on Wednesday.
   (b) There is a person who has run a marathon in every state in the United States of America.

9. Write the negation of each of the following statements (First write each statement symbolically, then negate the symbolic statement, and finally, translate the negation back into plain English).

   (a) Everyone who took their driver's exam today passed the exam.

   (b) Some people like bowling and tennis.

   (c) If everyone passed the exam then everyone studied for the exam.

10. Negate each of the statements (your answer should be in symbolic form)

   (a) $\forall x \forall y\, [((x > 0) \land (y < 0)) \to (xy \geq 0)]$

   (b) $\exists x \forall y \forall z\, [(F(x, y) \land G(x, z)) \to H(y, z)]$

   (c) $\forall x \exists! y\, [S(x, y) \lor \neg R(x, y)]$

11. Determine whether or not the given argument is valid by translating it into symbolic form and then building a truth table for the argument.

   (a) If I work hard every day then I will get a promotion at work.
   If I do not get a promotion at work then I will not be able to afford my house payment.
   I can afford my house payment.

   Therefore I work hard every day.

   (b) If gas is expensive and parking is inconvenient then I will take the bus to school.
   If I take the bus to school then I will not be able to take a night class.
   I am taking a night class.

   Therefore gas is not too expensive or parking is convenient.

12. Determine whether or not the given argument is valid by translating it into symbolic form and identifying its form as a known valid argument form or a known fallacy.

   (a) If I want to go out on Saturday night then I need to study for my exam during the afternoon.
   I do not want to go out on Saturday night.

   Therefore I do not need to study for my exam during the afternoon.

   (b) If I study for my exam Saturday during the afternoon then I can go out on Saturday night.
   I do not go out on Saturday night.

   Therefore I did not study for my exam Saturday afternoon.

   (c) If I do not study for my exam then I will not get a good grade on it.
   I got a good grade on my exam.

   Therefore I studies for my exam.

13. Prove that each of the following arguments are valid by constructing a 2-column proof.

   (a) If you are unhappy with the results of the election then the candidate you voted for did not win.
   If the candidate you voted for did not win, then she will run again next election.
   The candidate you voted for is not running next election

   Therefore you are happy with the election results

   (b) Everyone who is brave and intelligent majors in mathematics.
   Tony is not a math major.
   Tony is brave.

   Therefore Tony is not intelligent.

14. Write a paragraph proof for each of the following Theorems.

   (a) Show that the product of three odd numbers is odd.

   (b) Prove that if n is an integer and that $n^2 + 11$ is even, then n is odd.

   (c) Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$.

   (d) Show that the following three statements are all equivalent:
   (i) x is rational
   (ii) $\frac{x}{2}$ is rational
   (iii) $x + 1$ is rational.

15. Prove or Disprove: The product of two irrational numbers is irrational.