Math 310
Exam 3 - Practice Problems

1. Determine whether or not the following binary relations are equivalence relations. Be sure to justify your answers.
   (a) $\{(0,0),(0,3),(0,4),(1,1),(1,2),(2,1),(2,2),(3,0),(3,3),(3,4),(4,0),(4,3),(4.4)\}$ on the set $A = \{0,1,2,3,4\}$
   (b) $\{(a,a),(a,b),(b,a),(b,b),(b,d),(c,c),(d,b),(d,d)\}$ $A = \{a,b,c,d\}$
   (c) $\{(x,y) \mid y$ is a biological parent of $x\}$ on the set of all people.
   (d) $\{(x,y) \in \mathbb{N} \times \mathbb{N} \mid lcm(x,y) = 10\}$
   (e) $\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2 + 1\}$
   (f) for each of (a)-(e) that **are** equivalence relations, find the equivalence classes for the relation.

2. Define a relation R on $\mathbb{R}^2$ by $\{((x_1,y_1),(x_2,y_2)) \mid (x_1^2 + y_1^2) = (x_2^2 + y_2^2)\}$
   (a) Show that R is an equivalence relation.
   (b) Describe the equivalence classes of R.

3. Given that $A = \{0,1,2,3,4\}$
   (a) Find the smallest equivalence relation on A containing the ordered pairs $\{(1,1),(1,2),(3,4),(4,0)\}$
   (b) Draw the graph of the equivalence relation for found in part (a).
   (c) List the equivalence classes of the relation for found in (a).

4. For each of the following collections of subsets of $A = \{1,2,3,4,5\}$, determine whether of not the collection is a partition. If it is, list the ordered pairs in the equivalence relation determined by the partition.
   (a) $\{\{1,2\},\{3,4\},\{5\}\}$
   (b) $\{\{1,2,4\},\{3\},\{5\}\}$
   (c) $\{\{1,2,3,4\},\{5\}\}$
   (d) $\{\{1,2\},\{3\},\{5\}\}$
   (e) $\{\{1,2\},\{2,3,4\},\{5\}\}$

5. Determine whether or not the following binary relations are partial orders. Be sure to justify your answers.
   (a) $\{(0,0),(0,1),(0,2),(0,3),(0,4),(1,1),(1,2),(1,3),(1,4),(2,2),(2,4),(3,3),(3,4),(4,4)\}$ on the set $A = \{0,1,2,3,4\}$
   (b) $\{(a,a),(a,d),(b,b),(b,d),(c,c),(c,d),(d,d)\}$ on the set $A = \{a,b,c,d\}$
   (c) $\{(x,y) \mid y$ is a biological parent of $x\}$
   (d) $\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2 + 1\}$
   (e) for each of (a)-(d) that are posets, draw the Hasse diagram for the poset (for those that are defined on infinite sets, only draw a finite subpart of the diagram).

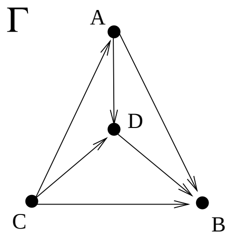
6. Indicate which element is greater for each given pair using the standard lexicographic ordering.
   (a) $(2,7)$ and $(3,4)$
   (b) $(2,7,4,9)$ and $(2,4,7,9)$
   (c) (a,c,e,d) and (i,c,e,d)
   (d) (b,a,n,d,a,n,a) and (b,a,n,a,n,a,s)

7. Draw the Hasse Diagram for the poset $(\mathcal{P}(\{1,2,3\}), \supseteq)$

8. Draw the Hasse Diagram for the poset $(\mathcal{P}(\{0,1,2,3\}), \subseteq)$

9. Given the poset $(\{1,2,3,5,6,7,10,20,30,60,70\}, |)$
   (a) Draw the Hasse Diagram for this poset.
   (b) Find the maximal elements.
   (c) Find the minimal elements.
   (d) Find the greatest element or explain why there is no greatest element.
   (e) Find the least element or explain why there is no least element.
   (f) Find all upper bounds of $\{2,5\}$
   (g) Find the least upper bound of $\{2,5\}$ (if it exists).
   (h) Find all lower bounds of $\{6,10\}$
   (i) Find the greatest lower bound of $\{6,10\}$ (if it exists).
   (j) is this poset a lattice? Justify your answer.

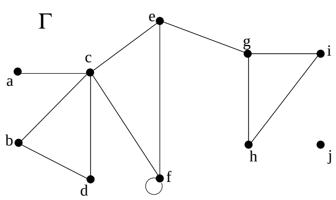
10. Suppose the following graph represents the result of a round robin volleyball tournament. Give the won lost record for each team who participated. Who won the tournament?



11. State what type of graph model you would use for each of the following. Make it clear what the vertices in your model represent, what the edges represent, whether the edges are directed or undirected, and whether not not loops and multiedges are allowed.

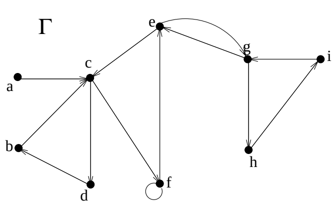
 (a) A graph model for the streets in Moorhead, Minnesota.
 (b) A graph that models the molecular structure of a certain protein molecule.
 (c) A graph modeling all the phone calls made on a certain carrier during the last hour.
 (d) A graph modeling a computer network.

12. Given the following undirected graph:



 (a) Find $|E|$
 (b) Find $|V|$
 (c) Find $deg(c)$, $deg(f)$, and $deg(g)$
 (d) List all isolated vertices of Γ
 (e) List all pendant vertices of Γ
 (f) Is Γ connected? Justify your answer. How many connected components does Γ have?
 (g) List all bridge edges in Γ.
 (h) List all cut vertices in Γ.
 (i) Find the set of vertices adjacent to vertex c.
 (j) Find the degree sequence for Γ.
 (k) Verify that the Handshaking Theorem holds for Γ.
 (l) Form the adjacency matrix for Γ with the vertices ordered alphabetically.
 (m) Form an incidence matrix for Γ with the vertices ordered alphabetically and the edges in an order of your choosing.

13. Given the following directed graph:



 (a) Find $|E|$
 (b) Find $|V|$
 (c) Find $deg^-(c)$, $deg^-(f)$, $deg^+(c)$, and $deg^+(f)$.
 (d) Is Γ strongly connected? Justify your answer. How many strongly connected components does Γ have?
 (e) Γ weakly connected? Justify your answer.
 (f) Find the set of vertices adjacent to vertex c.
 (g) Find the set of vertices adjacent from vertex c.
 (h) Form the adjacency matrix for Γ with the vertices ordered alphabetically.

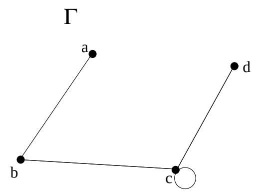
14. Use the Handshaking Theorem to prove that an undirected graph has an even number of vertices of odd degree.

15. (a) Draw each of the following graphs:

i. K_4	iv. K_6	vii. $K_{1,4}$
ii. C_8	v. Q_3	viii. $K_{3,2}$
iii. W_5	vi. C_7	ix. $K_{4,3}$

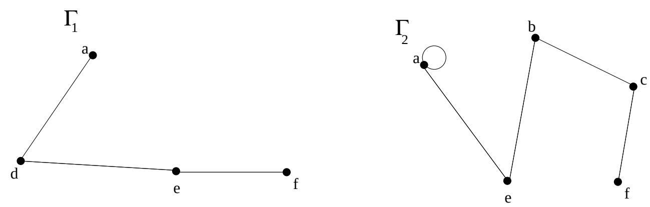
(b) Which of the graphs above are bipartite? Justify your answer.

16. Draw all subgraphs having all four vertices of the following graph.



17. From the list of subgraphs you found above, draw one representative of each isomorphism class.

18. Find the union of the following graphs:



19. 5 boys (Abe, Ben, Chuck, Donald, Elmer) and 6 girls (Francine, Gretchen, Heather, Irene, Jennifer, and Katie) are trying to find dates to the junior prom. Francine is willing to go with Ben, Chuck, Donald, or Elmer. Gretchen is willing to go with Abe or Donald. Heather is willing to go with Ben or Chuck. Irene is willing to go with Chuck or Donald. Jennifer is willing to go with Ben, Chuck, or Donald. Katie is willing to go with Donald or Ben.

(a) Draw a graph modeling this situation.

(b) Find a matching for which every boy has a date to the prom.

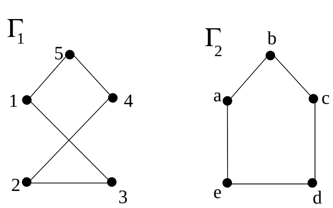
20. Draw a simple graph with each of the following degree sequences or state why one is not possible.

(a) 3, 3, 2, 2 (b) 4, 3, 2, 1 (c) 4, 3, 2, 2, 1

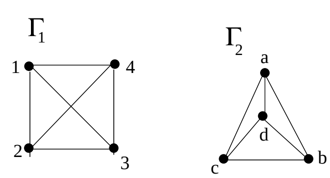
(d) 6, 2, 2, 2, 2, 1, 1 (e) 6, 2, 2, 1, 1

21. For each pair of graphs, prove that the graphs are isomorphic, or prove that they cannot be isomorphic.

(a)

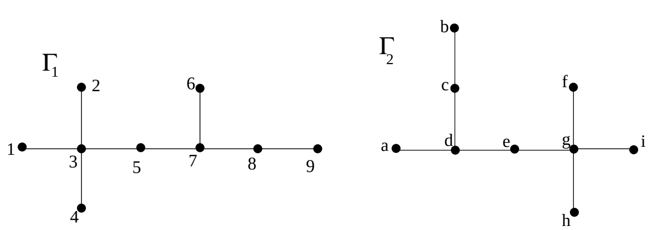


(b)



(c)

(d)



22. For each of the following graphs determine:

i) Whether or not the graph has an Euler Circuit.

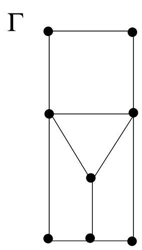
ii) Whether or not the graph has an Euler Path.

iii) Whether or not the graph has a Hamilton Circuit.

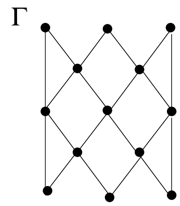
iv) Whether or not the graph has a Hamilton Path.

Be sure to justify your answers.

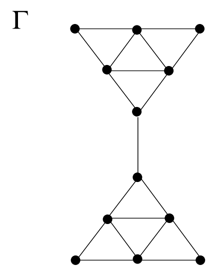
(a)



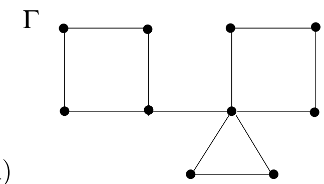
(b)



(c)



(d)

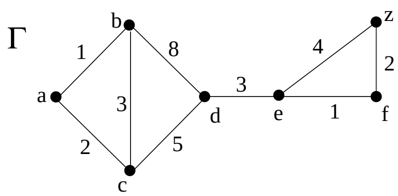


23. Draw a graph that satisfies the hypotheses of Dirac's Theorem. Explain how you know that each hypothesis is satisfied.

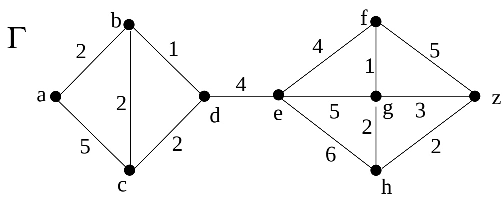
24. Draw a graph that satisfies the hypotheses of Ore's Theorem. Explain how you know that each hypothesis is satisfied. (your example may not be a complete graph.)

25. For each of the following weighted graphs, use Dijkstra's Algorithm to find a shortest path from a to z.

(a)

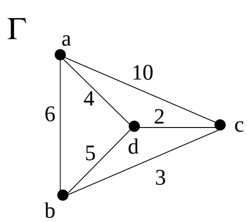


(b)



26. Use a brute force algorithm to find an optimal solution for the traveling salesman problem for the given graph starting at "home" vertex a:

(a)



(b)