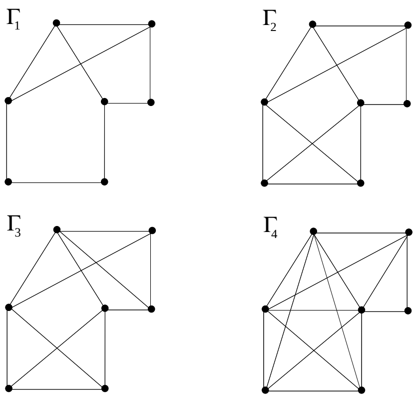
Math 310
Exam 4 - Practice Problems

1. Determine whether of not each of the following graphs is planar. If a graph is planar, exhibit a planar drawing of the graph and verify that Euler's formula holds for this representation of the graph. If a graph is not planar, provide an argument that proves that the graph cannot be planar.



2. Find the vertex chromatic number for each of the graphs given above. You do not need to prove your answer, but you should exhibit a coloring of the vertices of the graph that verifies your answer.
3. Find the edge chromatic number for the graphs in the previous problem.
4. Let $\Gamma = W_8$. Find the vertex chromatic number and the edge chromatic number of Γ. Provide a proof for your answer.
5. For each description given, either draw a planar graph that meets the description or prove that no planar graph can meet the description given.

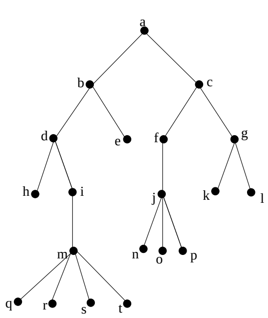
   (a) A simple graph with 5 vertices and 8 edges.

   (b) A simple graph with 6 vertices and 13 edges.

   (c) A simple bipartite graph with 7 vertices and 10 edges

   (d) A simple bipartite graph with 7 vertices and 11 edges.

6. Prove that every tree on n vertices has $n - 1$ edges.
7. Prove that an m-ary tree of height h has at most m^h leaves.
8. Answer the following questions based on the rooted tree shown below:



   (a) List the children of vertex j.

   (b) List the ancestors of vertex s

   (c) List the siblings of vertex q

   (d) Find the number of leaves in this rooted tree.

   (e) List all level 3 vertices in this rooted tree.

   (f) Find the least m for which this tree is a rooted m-ary tree.

   (g) Find the height of this rooted tree.

   (h) Find the order that you would visit the vertices of this tree if you use postorder traversal to visit the vertices.

   (i) Find the order that you would visit the vertices of this tree if you use preorder traversal to visit the vertices.

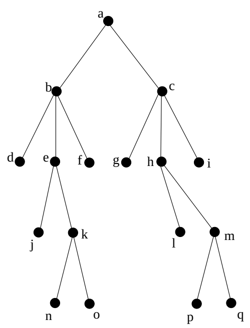
   (j) Find the order that you would visit the vertices of this tree if you use inorder traversal to visit the vertices.

9. A chain letter starts when a person sends a letter to 5 people. Each person who sends the letter to 5 other people who have never received it or does not send it to anyone. Suppose that 10,000 people send out the letter before the chain ends and that no one receives more than one letter. How many people receive the letter? How many people do not send it out?

10. (a) Draw a binary search tree for the sentence "Now is the time for all good men to come to the aid of their country."
    (b) How many comparisons are needed to locate the word *time* in this tree?
    (c) How many comparisons are needed to add the word *waffle* to this tree?

11. How many weighings are needed to find *two* counterfeit coins among 5 total coins, one heavier than a normal coin and the other lighter than a normal coin. Give an algorithm that proves your answer.

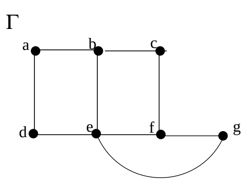
12. Consider the following tree:



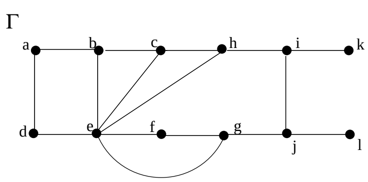
    (a) List the vertices in the order that you would visit them if you traversed the tree in preorder.
    (b) List the vertices in the order that you would visit them if you traversed the tree in inorder.
    (c) List the vertices in the order that you would visit them if you traversed the tree in postorder.

13. Consider the expression: $(3 - 4 \cdot 2)^2 - (5 + (5 - 2)^3)$
    (a) Draw the rooted tree representing this computation.
    (b) Write this expression in prefix notation.
    (c) Write this expression in postfix notation.

14. Draw all possible spanning trees for the following graph:

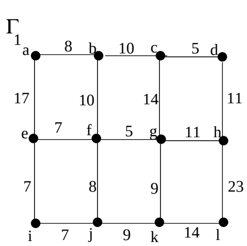


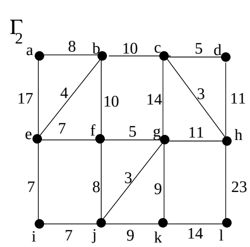
15. Given the following graph:



    (a) Use a depth first search to find a spanning tree for this graph starting at root vertex a.
    (b) Use a breadth first search to find a spanning tree for this graph starting at root vertex e

16. For the weighted graphs given below:





    (a) Use Prim's Algorithm to find a minimum spanning tree for each graph.
    (b) Use Kruskal's Algorithm to find a minimum spanning tree for each graph.

17. Describe an algorithm to determine whether or not a simple connected graph has a Hamilton Circuit using depth first searches.