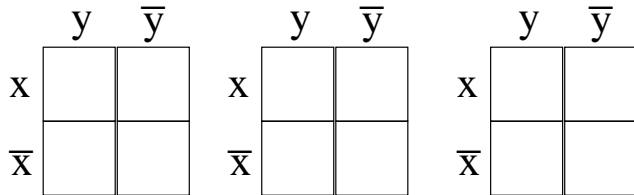


Karnaugh Maps:

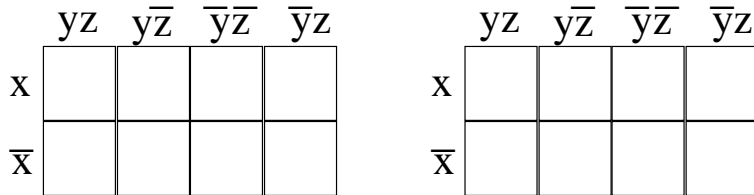
A **Karnaugh Map** is a diagrammatical method for finding the minimization for a Boolean expression in disjunctive normal form. To use a K-map to simplify an expression, we put a 1 in the square in the diagram corresponding to each minterm represented in the Boolean expression. We circle all *blocks* in the diagram. Blocks are collections of 2^k adjacent cells for some k . A minimization for the given Boolean expression is found by choosing a collection of *maximal* blocks which *cover* the 1s in the diagram. A minimal expression is found by writing out the terms associated with the collection of blocks chosen.

Examples:

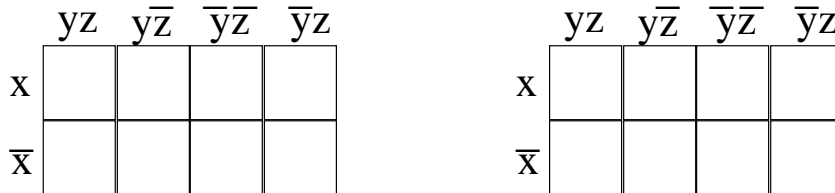
1. $x\bar{y} + \bar{x}y$ 2. $xy + \bar{x}\bar{y}$ 3. $xy + \bar{x}y + \bar{x}\bar{y}$



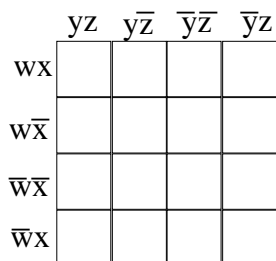
4. $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z}$ 5. $x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$



6. $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$ 7. $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$



8. $wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z$



Method 2: The Quine-McClusky Method

K-Maps become very difficult to draw and interpret once we have more than 4-5 variables in our expression. The Quine-McClusky method is an algorithmic procedure that automates the process of finding a minimization of a Boolean Expression. The algorithm proceeds as follows:

1. List each minterm in the given Boolean expression one at a time and convert each minterm to a binary string.
2. Find all pairs of minterms that can be combined into a shorter term (the binary strings for these terms will differ in exactly one position). List these pairs and their strings in a new column.
3. Apply the same process in your new column of shortened strings.
4. Continue this process until all possible terms have been combined. This process results in a list of terms of various length.
5. Work backwards from the shortest terms you found up to the largest terms choosing a minimal collection of terms to generate all of the original minterms (it may be useful to create a separate table for this step).

Examples: