Math 310
Project 1 Handout
Due: Tuesday, April 1st by 4:00pm

Instructions: This project is designed to give you an opportunity to explore some of the concepts from class in a little more depth. You may work with at most one other student on this assignment. If you decide to work with another student, you may turn in a combined paper with both your names listed.

1. (12 points) In $(1, 2, 3) - \textit{Misère NIM}$, the game begins with a pile of N stones. On their turn, a player can take either 1, 2, or 3 stones. However, unlike the version presented in class, in this case, the player that takes the last stone **loses** the game.

   (a) Which player has a winning strategy if the game starts with 7 stones?

   (b) Do a complete analysis of *Misère NIM* as the game is defined above. That is, determine which player has a winning strategy for any value of N [Hint: split into cases].

   (c) Generalize your results to $(1, 2, 3, ..., k) - \textit{Misère NIM}$ in which a player can take either 1, 2, 3, ..., or k stones.

2. (10 points) This part of the project involves a famous set-theoretic problem called Russell's Paradox. According to Russell's definitions, a set A is called **normal** if A is *not* an element of itself. Similarly, a set is **abnormal** if it *is* an element of itself.

   (a) Give an example of a set that is normal.

   (b) Give an example of a set that is abnormal.

   (c) Let $\mathcal{N} = \{A \mid A \text{ is a set that is normal }\}$ and let $\mathcal{A} = \{A \mid A \text{ is a set that is abnormal }\}$.
   Is $\mathcal{N}$ a normal set or an abnormal set? How about $\mathcal{A}$? Explain how this leads to a paradox and comment on what caused things to go wrong.

3. (10 points) The Well Ordering Principle states the following: "Every non-empty set of non-negative integers has a least element".

   This question involves another famous paradox called the Berry Paradox.

   (a) Use the well ordering principle to prove the following statement: "Every positive integer can be described using no more than fifteen English words." In your proof, you may assume that we are taking words from a particular dictionary that has a compete list of all acceptable words. [Hint: Suppose, to obtain a contradiction, that there are positive integers that cannot be described using no more than fifteen words. Consider the smallest such number, which exists by the Well Ordering Principle. This positive integer can be described as: "the smallest positive integer that cannot be described using no more than fifteen words"]

   (b) Next, give an argument that shows that since the number of words in our dictionary is finite, then there are a finite number of descriptions involving fifteen words or less. Combine this with the fact that there are an infinite number of non-negative integers to argue that there are positive integers that cannot be described using no more than fifteen English words.

   (c) Which of these proofs do you find more believable and why? Can you think of a way to resolve this paradox?