Math 310
Set Proof Example

Example: Let A, B, and C be sets. Show that $A \cap (B - C) = (A \cap B) - C$ using a paragraph proof that demonstrates that each side is a subset of the other side.

First, suppose that $x \in A \cap (B - C)$. Then, using the definition of set intersection, $x \in A$ and $x \in (B - C)$. Using the definition of set subtaction, this means that $x \in A$, $x \in B$, and $x \notin C$. Since $x \in A$ and $x \in B$, then $x \in (A \cap B)$. Moreover, we still have that $x \notin C$. Hence $x \in (A \cap B) - C$.

Since we have shown that an arbitrary element of $A \cap (B - C)$ is also an element of $(A \cap B) - C$, then $A \cap (B - C) \subseteq (A \cap B) - C$.

Next, suppose that $x \in (A \cap B) - C$. Then, using the definition of set subtraction, $x \in (A \cap B)$ and $x \notin C$. Using the definition of set intersection, since $x \in (A \cap B)$, then $x \in A$ and $x \in B$. Since $x \in B$ and $x \notin C$, then $x \in (B - C)$. Moreover, we still have that $x \in A$. Hence $x \in A \cap (B - C)$.

Since we have shown that an arbitrary element of $(A \cap B) - C$ is also an element of $A \cap (B - C)$, then $(A \cap B) - C \subseteq A \cap (B - C)$.

Thus $A \cap (B - C) = (A \cap B) - C$

$\square$