

1. For each of the following series, determine whether the series converges or diverges.

$$(a) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n-1}{n^2}$$

$$(c) \sum_{n=1}^{\infty} (-1)^{n+1} e^{\frac{1}{n}}$$

$$(b) \sum_{n=1}^{\infty} (-1)^n \frac{n}{2n-5}$$

$$(d) \sum_{n=1}^{\infty} (-1)^{n+1} \left( \frac{\ln n}{n} \right)$$

2. Estimate the sum of the following series to within two decimal places of accuracy

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{10^n}$$

$$(b) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{7}{n^5}$$

3. Determine how many terms you would need in order to estimate the following sums to within .0001 (you do **not** need to find an estimate)

$$(a) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{\sqrt{n}}$$

$$(b) \sum_{n=1}^{\infty} (-1)^n \frac{4^n}{n^n}$$

4. Determine whether each of the following series are absolutely convergent, conditionally convergent, or divergent:

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{2}{3^n}$$

$$(c) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1}$$

$$(b) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3n}{500n+1}$$

$$(d) \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

(e)  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

(f)  $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n!}$

5. The following is a proof that  $2=1$ . Consider the alternating harmonic series  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ . We know that this series converges conditionally. Assume the sum of the series is  $S$ .

(a) Write out the first 14 terms of this series. We've done the first 3.

$$S = 1 - \frac{1}{2} + \frac{1}{3} -$$

(b) Multiply what you have in part (a) by 2 and simplify your fractions:

$$2S =$$

(c) Collect terms with the same denominator in part (b):

$$2S =$$

(d) Rearrange what you have in part (c) so that you have the series you began with:

$$2S = 1 - \frac{1}{2} +$$

(e) Thus:

$$2S = S \text{ and dividing by } S \text{ we get } 2 = 1.$$

(f) We know that  $2 \neq 1$ . What does your work above tell you about conditionally convergent series?