1. For each of the following power series, find the interval of convergence and the radius of convergence of the series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!} x^n$

   Interval of Convergence: ____________

   Radius of Convergence: ____________

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} x^n$

   Interval of Convergence: ____________

   Radius of Convergence: ____________

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{10^{n+1}}{3^{2n}} x^n$

   Interval of Convergence: ____________

   Radius of Convergence: ____________

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}(2x-3)^n$

Interval of Convergence: ____________

Radius of Convergence: ____________

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln n}(x-\sqrt{2})^n$

Interval of Convergence: ____________

Radius of Convergence: ____________

2. Recall that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ if $|x| < 1$. Use this fact to determine a series representation for the following. Find the radius of convergence for these.

   (a) $\dfrac{1}{1-2x}$ (b) $\dfrac{1}{1+x^2}$ (c) $\dfrac{1}{3-5x}$

3. The series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$$

   converges to $\sin(x)$ for all real numbers x.

   (a) Use differentiation to find the first six terms of a series for $\cos(x)$.

   (b) Now, try finding a series for $\cos(x)$ by computing $-\int \sin(x)dx$.

   (c) Did you get the same answer? Explain what is going on.

   (d) Use the fact that $\cos(0) = 1$ to determine which of these is correct.

4. We know that $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$.

(a) Differentiate both sides, then multiply by x and find the interval of convergence for this new series.

(b) Use the result from part (a) to find the value of $\sum_{n=0}^{\infty} \dfrac{n}{2^n}$.

(c) Use the concept from parts (a) and (b) to find the value of $\sum_{n=0}^{\infty} \dfrac{n^2}{3^n}$.